NATURAL SCIENCES TRIPOS Part II

Thursday 23 May 1985 1.30 to 4.30

PHYSICS AND THEORETICAL PHYSICS (3)

Answer **four** *questions, including at least* **one** *from* **each** *Section.*
The answers to Sections A, B and C should be tied up in separate bundles. Even if no question is answered in a Section, a cover-sheet for that Section must be completed.
This paper contains **four** *sides.*
This paper is accompanied by a sheet giving the values of constants.

SECTION A

1 A particle of mass m is moving in a one-dimensional harmonic potential $V(q) = \frac{1}{2}m\omega^2q^2$. Define an operator $A = (2m\hbar\omega)^{-\frac{1}{2}}(P + imωQ)$, where P and Q are momentum and position operators for the particle.

(a) Show that $AA^+ = H/\hbar\omega - \frac{1}{2}$, $A^+A = H/\hbar\omega + \frac{1}{2}$, $[H,A] = +\hbar\omega A$, $[H,A^{*}] = -\hbar\omega A^{*}$, where H is the Hamiltonian operator.

(b) Hence show that the sequence of energy eigenvalues has the form $(n+\frac{1}{2})\hbar\omega$, where n is a positive integer or zero.

(c) A perturbing potential is superimposed of the form λq^4. Calculate the shift in the energy levels to first order in λ.

2 Write notes on *three* of the following, with special emphasis on experimental observations:

(a) measurement of two physical quantities in a single quantum system;

(b) zero point energy;

(c) the Pauli exclusion principle;

(d) quantum-mechanical tunnelling.

3 Discuss coupling of angular momenta of electrons in multi-electron atoms, paying attention to the physical principles rather than mathematical details.

Derive the possible Russell-Saunders (LS) states for two p electrons having *different* principal quantum numbers. Show that when the principal quantum numbers are *the same*, the possible states are:

$$^1S_0 \qquad ^3P_{2,1,0} \qquad ^1D_2$$

For the latter case (i.e. when the principal quantum numbers are the same) obtain also the possible states in the limit of *jj* coupling.

[TURN OVER for continuation of Question 3

Wavelengths of two transitions between ^3P and ^1D states can be assigned in various atoms and ions as:

C	$(1s^2\ 2s^2\ 2p^2)$	λ = 985 and 983 nm
Sn	$(1s^2\ 2s^2\ 2p^6 \ldots\ 5p^2)$	λ = 1.93 and 1.44 μm
Cr^{18+}	$(1s^2\ 2s^2\ 2p^2)$	λ = 98 and 73 nm .

Comment on these data and on the observation that the transitions in carbon are very weak while those in Cr^{18+} are quite strong.

SECTION B

4 An experiment is carried out to detect solar neutrinos using the reaction

$$\nu + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$$

in which 0.2 atoms day^{-1} of ^{37}Ar were produced in 400 m^3 of tetrachloroethene (C_2Cl_4). Theories of solar neutrino production suggest a flux at the earth's surface of 10^{13} m^{-2}s^{-1} neutrinos energetic enough to cause the reaction. Estimate the cross-section per ^{37}Cl nucleus for such neutrinos.

The beta-decay process of the ^{36}Cl isotope is observed, and the maximum energy of the decay electron is found to be 712.0 keV. What information does this give on the neutrino rest mass? Summarize the physical arguments from beta-decay which suggest the existence and properties of the neutrino.

[$(M-A)/m_u$: of nuclides ^{36}Cl, -0.031 691; ^{36}Ar, -0.032 456.
m_uc^2 = 931.502 MeV. The isotopic abundance of ^{37}Cl is 25%.
Density of tetrachloroethene = 1.5 Mg m^{-3}.]

5 Describe the essential features of a thermal nuclear reactor, making clear the physical bases of the relevant phenomena.

6 Write briefly about *two* of the following topics:

(a) resonance absorption;
(b) the properties of the nuclear force that may be deduced from measurements on the deuteron;
(c) the concept of refractive index as applied to the behaviour of thermal neutrons.

SECTION C

7 Account for *three* of the following observations in as much detail as possible:

(a) The decay $\pi^0 \rightarrow 2\gamma$ has a half-life of 8.10^{-17} s whereas the decay $\pi^+ \rightarrow \mu^+\nu$ occurs with a half-life of 26 ns.

(b) The reaction $\pi^- + d \rightarrow n + n + \pi^0$ cannot take place for pions which have been brought to rest in a deuterium bubble chamber.

(c) Despite the Pauli principle, the Δ^{++} baryon which has spin 3/2 is thought to consist of a bound state of three u quarks having zero relative orbital angular momentum. colour part of wave function

(d) The ψ-meson of mass 3.1 GeV/c^2 appears in the reaction $e^+e^- \rightarrow$ hadrons as a resonant state of extremely narrow width (63 keV).

8 Write brief notes on *three* of the following:

(a) proton decay and its significance for the unification of the electroweak and strong forces;

(b) the classification of baryons in the quark model;

(c) photon exchange and gluon exchange : similarities and differences;

(d) gauge symmetries.

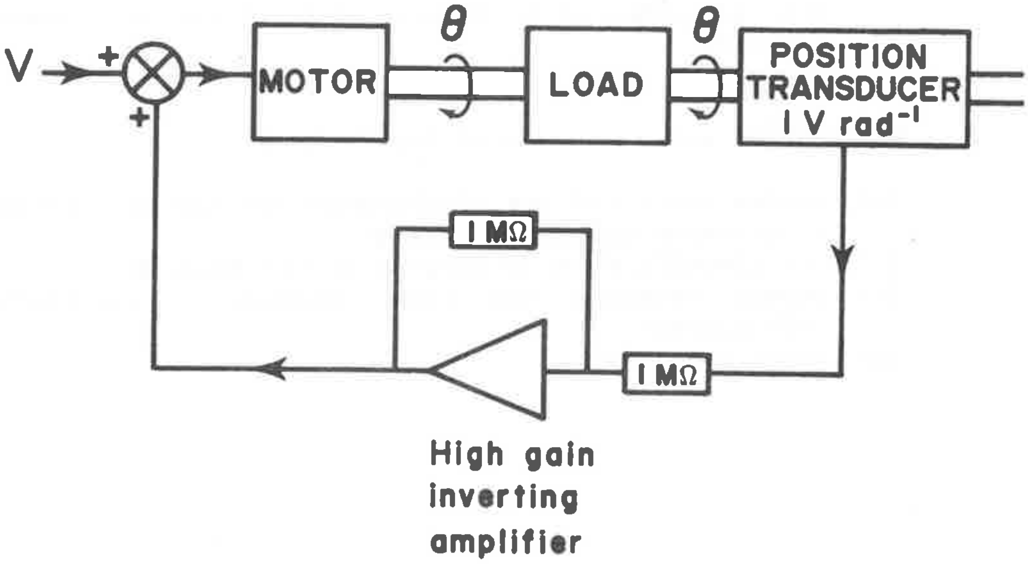
[TURN OVER

9 Why is it sometimes necessary to add velocity feedback to a position control system? What are the advantages of this method of modifying the performance of a system?

The position servo shown below has the following parameters:

Motor: resistance 10 Ω
back e.m.f. $1.5(\dot{\theta}/\text{rad s}^{-1})$ V
torque constant 25 N m A^{-1}

Moment of inertia of load plus motor 40 kg m^2
Sensitivity of position transducer 1 V rad^{-1}



Derive an expression for the transfer function, $\Theta(s)/V(s)$ of the system with the feedback shown, where s is the complex frequency.

Show that the system can be made critically-damped by including an additional feedback voltage proportional to velocity, $k\dot{\theta}$ where k = 6.5 V s rad^{-1}.

How might the feedback loop be modified in order to include this extra term?