NATURAL SCIENCES TRIPOS Part II

Friday 24 May 1985 9 to 12

PHYSICS AND THEORETICAL PHYSICS (4)

You are required to write a prepared essay **either** *on this paper* **or** *on Paper 5. The subject of your essay may be chosen from the titles previously announced (which are listed in this paper and will be set out again in Paper 5). Alternatively, if you have been given the examiners' approval, you may write on a subject of your own choice. Write at the head of your essay its title and the letter indicating the group to which it has been assigned by the examiners. The essay carries marks equivalent to half a paper.*

If you write your essay on this paper, you are required to answer **two** *further questions, unless you have submitted project work in substitution for the second half of this paper, when you will be required to leave after 1½ hours.*

If you intend to write your essay on Paper 5, you are required to answer **four** *questions on this paper, unless you have submitted project work in substitution for the second half of this paper, when you should answer* **two** *questions and will be required to leave after 1½ hours.*

You may **not** *answer questions on the option associated with your prepared essay.*

You **must** *write at least* **one** *answer related to a major option either on this paper or on Paper 5. This answer must consist* **either** *of a question answered for a major option on this paper* **or** *of an essay assigned to a major option and written either on this paper or on Paper 5.*

Section I (Major options) will be found on pages 2 to 11 and Section II (Minor options) on pages 12 to 18.

This paper is accompanied by a sheet giving the values of constants.

Please tie up the essay and the answers to **each** *group of questions with a separate cover-sheet.*

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SECTION I Major Options

Group A ELECTRONS IN SOLIDS (Major Option)

Essay Title: Fermi surfaces and their relevance to the physical properties of metals and semiconductors.

Questions:

A1 Consider a crystal with simple cubic unit cell containing one atom.

(a) Show that the tight-binding approximation leads to an energy band for an atomic 1s state of the form

$$E(\mathbf{k}) = E_0 - \alpha - 2\beta\ (\cos k_x a + \cos k_y a + \cos k_z a)$$

where E_0 is the eigenvalue of the unperturbed atomic state, a is the lattice parameter and $\mathbf{k}$ the electron wavevector. State clearly any assumptions you make and explain the significance of the quantities α and β.

(b) Sketch the variation of energy with $\mathbf{k}$ between $\mathbf{k} = (0,0,0)$ and the Brillouin zone boundaries along the symmetry directions [100], [110] and [111].

(c) Sketch the form of the wavefunction along lines of atoms in the x, y and z directions for the states

$$\mathbf{k} = (0,0,0),\quad (1,0,0)\pi/a,\quad (1,1,0)\pi/a,\quad (1,1,1)\pi/a\ .$$

Show that features of these wave functions are consistent with their relative energies.

(d) Determine the effective mass for electrons near the (0,0,0) and (1,1,1) states for $\beta = 1$ eV and $a = 0.1$ nm.

(e) Describe the total density of states curve for the energy band, giving as much quantitative information as possible.

A2 Explain the physics involved in the formation of an inversion layer at a surface of a p-type semiconductor.

Show that the potential energy of an electron near a surface that is just inverted is given by

$$V(z) \approx (2N_a E_g/\epsilon_0 \epsilon_r)^{1/2}\, z$$

where z is depth, N_a the acceptor concentration, E_g the band gap, ϵ_0 the permittivity of free space and ϵ_r the relative permittivity of the semiconductor.

Obtain approximate expressions for the energies of the lowest two states for motion in the z-direction in the potential well at the surface. Hence estimate for silicon, with $N_a = 10^{22}\ \mathrm{m}^{-3}$, $E_g = 1.2$ eV, $\epsilon_r = 12$ and $m_e^*/m_e = 1$, the temperature below which electrons in an inversion layer behave as a two-dimensional electron gas.

Suggest ways in which you might expect to be able to demonstrate that the electron gas does become two-dimensional at low temperatures.

A3 Write briefly about *three* of the following:

(a) electron screening in metals;
(b) Jones zones and their application to the band structure of semiconductors such as silicon;
(c) the variation with electric field strength of conduction current in a semiconductor;
(d) structural phase transitions in metal alloys induced by changing electron/atom ratios.

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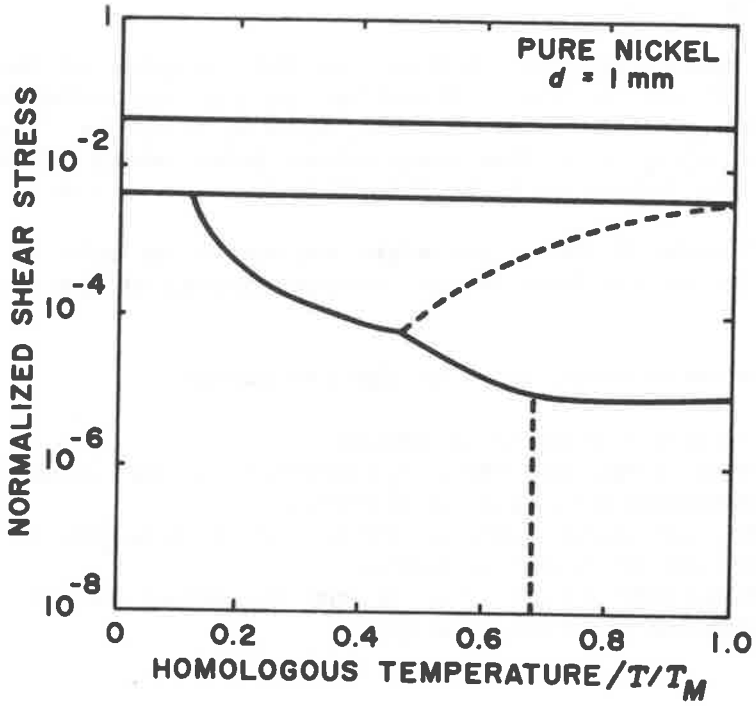
Group B MATERIALS (Major Option)

Essay Title: The glass transition in inorganic and polymeric materials.

Questions:

B1 The diagram shows a creep deformation map for nickel. Describe the mechanisms which operate for each of the regions, and discuss the value of such maps.



Obtain an approximate expression for the creep strain rate $\dot{\epsilon}$ when bulk diffusion dominates in terms of the coefficient for self-diffusion D_{sd}, the pressure p, the activation volume V, the temperature T and the grain size d. Comment on the role of grain size in determining the creep resistance of a material.

B2 Discuss *three* of the following:

(a) diffusion in polymers;
(b) mechanisms of ductile fracture;
(c) stability of colloids;
(d) inhomogeneous deformation of amorphous alloys.

B3 Figure I shows data for the dependence of dislocation velocity on applied stress. Discuss the mechanisms which control the shape of such curves.

[Question B3 continues on page 5

Fig. I

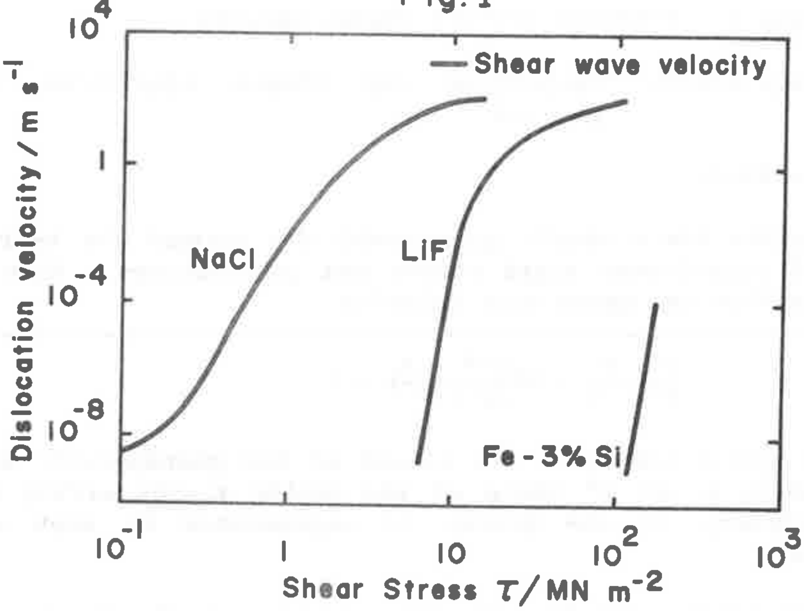
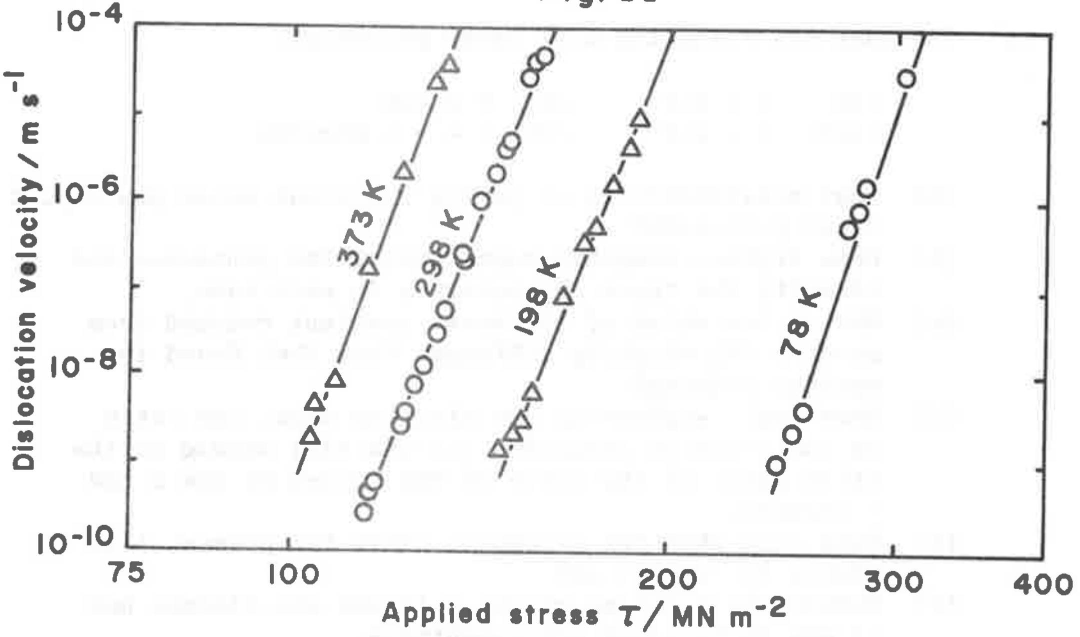


Figure II gives data for the effect of the applied stress on dislocation velocity for silicon iron.

Fig. II



Some authors have suggested that at low dislocation velocities

$$\tau = \tau_0 \exp\{(E - \tau V)/kT\}$$

where τ is the applied stress, E is the activation energy, V the activation volume, T the temperature and τ_0 a constant for a particular dislocation velocity. Obtain values for E, V and τ_0 for a dislocation velocity of 10^{-8} m s^{-1} and comment on their magnitudes.

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Group C PARTICLE PHYSICS (Major Option)

Essay Title: Symmetries and broken symmetries in particle physics.

Questions:

C1 Explain how a simple quark model can account for hadron masses in terms of constituent quark masses and interactions. Show that such a model implies the meson mass relation

$$\frac{\rho - \pi}{K^* - K} = 2\left(\frac{3K^* + K}{3\rho + \pi}\right) - 1$$

where π and K represent the masses of the pseudoscalar mesons $\pi(135)$ and K(494), ρ and K^* those of the vector mesons $\rho(770)$ and K*(892). What property of the quarks is represented by each side of the equation?

Investigate and discuss the validity of the above mass relation and the analogous one in which K and K* are replaced by the corresponding charmed mesons D(1865) and D*(2010).

C2 Consider the following weak decay processes:

(i) $\mu \to e\nu\bar{\nu}$ (ii) $\tau \to e\nu\bar{\nu}$
(iii) $\tau \to \mu\nu\bar{\nu}$ (iv) $\tau \to \nu$ + hadrons.

(a) What manifestations of parity violation would you expect in these processes?
(b) Draw Feynman diagrams representing the processes and identify the types of neutrinos in each case.
(c) Why is the value of the Fermi constant deduced from process (i) slightly different from that found in nuclear β-decay?
(d) Show that, neglecting the electron mass, the ratio of the rates of processes (i) and (ii) should be the fifth power of the ratio of the masses of the μ and τ leptons.
(e) Show also that the branching ratio for process (ii) should be roughly 20%.
(f) Deduce the lifetime of the τ lepton and discuss how it may be measured experimentally.

[mass of μ lepton, m_μ = 106 MeV/c^2;
mass of τ lepton, m_τ = 1784 MeV/c^2;
lifetime of μ lepton, $\tau_\mu = 2.2 \times 10^{-6}$ s.]

C3 Write notes on *three* of the following:

(a) testing time-reversal invariance;
(b) the detection of charged particles;
(c) deep inelastic neutrino scattering;
(d) running coupling constants.

Group D OBSERVATIONAL COSMOLOGY (Major Option)

Essay Title: The cosmological significance of observations at radio frequencies.

Questions:

~~D1~~ The Robertson-Walker metric can be written in the form

$$ds^2 = dt^2 - \frac{\mathbb{R}^2(t)}{c^2}\{\, dr^2 + R^2 \sin^2(r/R)\ (d\theta^2 + \sin^2\theta d\phi^2)\} \ .$$

What are the physical meanings of $\mathbb{R}(t)$, R and cosmic time t?

If photons emitted at time t are received with a redshift z, show that $\mathbb{R}(t) = (1 + z)^{-1}$.

A Friedmann model is such that

$$\frac{d\mathbb{R}}{dt} = H_0(1 + z)^{½} \text{ and } R = \infty$$

where H_0 is the present value of Hubble's constant. Show that the angle $\Delta\theta$ subtended by a spherical galaxy of diameter L having a redshift z, is

$$\Delta\theta = (LH_0/2c)(1+z)[1-(1+z)^{-½}]^{-1} \ .$$

To what extent are observed angular diameters of value in the determination of cosmological parameters?

?. D2 A radio telescope consists of a parabolic reflector 50 m in diameter at the focus of which is mounted a receiver having a system temperature of 50 K and a bandwidth of 400 MHz centred at 15 GHz. How could this be used to search for anisotropies in the cosmic microwave background and what sensitivity might be achieved?

√ Discuss the importance of any anisotropies that might be expected on both large and small angular scales. To what extent have such anisotropies been observed?

~~D3~~ Write notes on *four* of the following:

~~(a)~~ the age of the Universe;
(b) the luminosity-volume test;
~~(c)~~ the origin of light elements;
~~(d)~~ the measurement of astronomical distances;
~~(e)~~ solar neutrinos.

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Group E PHYSICS OF THE EARTH AS A PLANET (Major Option)

Essay Title: Heat flow from the interior of the Earth.

Questions:

E1 Explain the terms *transform fault, fracture zone.*

Outline, without mathematical details, the derivation of the relationship between ocean depth and age of the crust.

Account for the change in depth across a fracture zone, giving a specific example.

The crust along the inactive part of a fracture zone is often found to have low seismic velocities. Why should this be so?

E2 Discuss reasons for low seismicity:

(a) in the lower crust;
(b) at great depth in certain subduction zones.

In case (b) use a simple order-of-magnitude argument to find a relationship between the subduction rate normal to the trench and the depth of the deepest earthquakes.

Use your relationship to discuss the variation of the depths of the deepest earthquakes along the Tonga-Kermadec subduction zone.

[For the lithosphere take density ρ = 3 Mg m^{-3}; specific heat capacity c_p = 1 kJ $kg^{-1}K^{-1}$; thermal conductivity λ = 4 J $K^{-1}m^{-1}s^{-1}$. The pole for relative motion between the Australian and Pacific plates is near 58°S, 168°E and the relative angular velocity of the two plates is 12.3 10^{-7}°$year^{-1}$.]

E3 Describe two major features of the *geomagnetic secular variation.*

State *Alfvén's Theorem* for a perfectly electrically conducting fluid, and explain its use in determining fluid motion at the core surface.

The radial component of a magnetic field at the core surface (r =c) obeys the equation

$$\dot{B}_r + \frac{1}{c\sin\theta}\left[\frac{\partial}{\partial\theta}(v_\theta B_r \sin\theta) + \frac{\partial}{\partial\phi}(v_\phi B_r)\right] = 0 \qquad (1)$$

where a dot denotes differentiation with respect to time and v is the velocity of the core fluid.

[Question E3 continues on page 9

Show that necessary conditions to be satisfied by B_r and the secular variation $\dot{B}_r$ have the form

$$\int_{S_i} \dot{B}_r \, dS = 0 \qquad (2)$$

and define the surfaces $\{S_i\}$.

Observations are made of $\dot{B}_r$ and B_r, and they are found to satisfy conditions (2). If $v_\theta = 0$, obtain a formula for v_ϕ. Show that the secular variation must satisfy the condition

$$\int_{\phi(A)}^{\phi(B)} \dot{B}_r d\phi = 0 \ ,$$

where the integral is along lines of latitude and A, B, are points where $B_r = 0$.

Use your formula to obtain an order of magnitude estimate of v_ϕ at the equator if B_r = 0.5 mT, $\dot{B}_r$ = 1000 nT/year and c = 3500 km. How does this estimate compare with the westward drift.

Group F MICROCOMPUTING IN PHYSICS (Major Option)

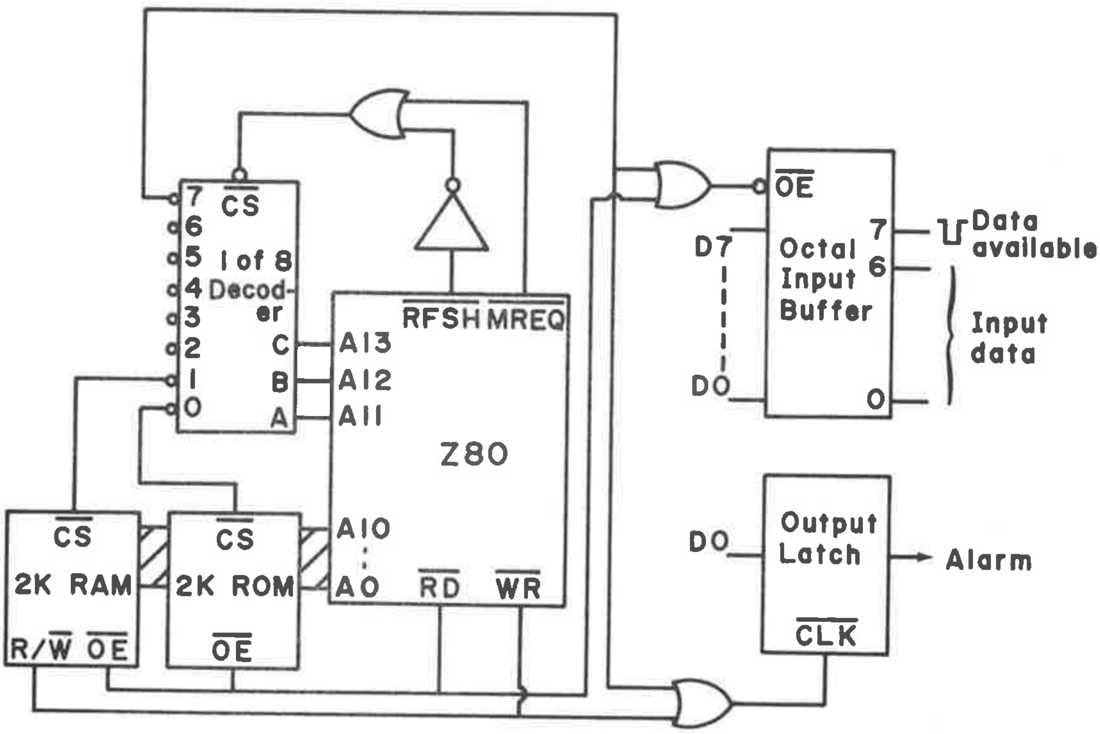
Essay Title: The laboratory use of microprocessors for controlling experiments, and for taking and analysing data.

Questions:

F1 Describe the basic components required to construct a microcomputer using a Z80 CPU.

A Z80 microprocessor is to be used for validating a sequence of 7-bit data bytes. The data consist of blocks of 254 values followed by two further 7-bit numbers (most significant - least significant) representing the 14-bit sum of the preceding 254 values. A square-wave of frequency approximately 1 kHz is provided to serve as a 'Data Available' signal. The system is required to output a signal and to 'call' (at a specified address) an alarm routine whenever an incorrect sequence is received. Deduce from the diagram below the relevant addresses for the system and write an Assembler program, starting at address zero, which will perform the required function.

[The two-input gates perform an OR function for positive inputs. Connections to the Z80 data bus are not shown.]



F2 It is required to record and control the temperature of a small experimental enclosure which is subject to a changing heat input. The enclosure contains a simple DC heater and a small variable-speed fan which draws in cooler external air. There are internal and external resistance thermometers. With the aid of block diagrams, outline the hardware of a Z80-based control system. Describe the software required to maintain the temperature at a specified value above ambient and to record, every second, the actual temperature of the enclosure with a resolution of better than 0.1 K. How might these data be displayed and stored permanently?

If it is required to cycle the enclosure through a sequence of temperatures at specified times, how might this be achieved?

F3 Write notes on *three* of the following:

(a) What are the problems of doing arithmetic calculations with a Z80 microprocessor? Consider the consequences of the instruction set and of possible number representations on both speed and precision.

(b) What are the minimum features required of an Assembler? Discuss the additional features which might be provided and the situations in which they could be of particular value.

(c) Explain the factors which must be considered when digitally sampling a continuous signal. Discuss briefly the basis of signal filtering as applied to a sampled digital signal, illustrating your answer with the case of a simple low-pass filter.

(d) Describe the various types of read-write and read-only memory commonly used in conjunction with microprocessors and give examples of applications for which each is best suited.

(e) Explain the purpose of interrupts and describe the ways in which they can be employed in the case of a Z80 microprocessor. What is necessary in order to allow the use of 'nested' interrupts?

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SECTION II Minor Options

Group G ADVANCED QUANTUM THEORY (Minor Option)

Essay Title: Coupling between electrons, phonons and the electromagnetic field.

Questions:

G1 Discuss the interpretation of the Schrödinger wavefunction, $\Psi(\boldsymbol{r},t)$, for a non-interacting electron, in terms of field operators $\hat{\Psi}(\boldsymbol{r},t)$, giving both the Hamiltonian and the single electron statefunction in second quantized form.

Show that $\hat{\Psi}(\boldsymbol{r},t)$ obeys the Schrödinger equation in a suitable representation, which you should explain.

Contrast carefully the way pairwise electron-electron interactions would be represented in a Hamiltonian in terms of field operators as opposed to a coordinate representation, and outline how the former can be interpreted in terms of a boson mediated interaction.

G2 Compare the Klein-Gordon and Dirac equations as Lorentz-covariant equations that describe relativistic particles. Your discussion should include reference to:

(a) the conservation of particle number;
(b) the angular momentum of a particle in a central field;
(c) the relationship between vector potential, and momentum and velocity of a charged particle.

Group H CHEMICAL PHYSICS (Minor Option)

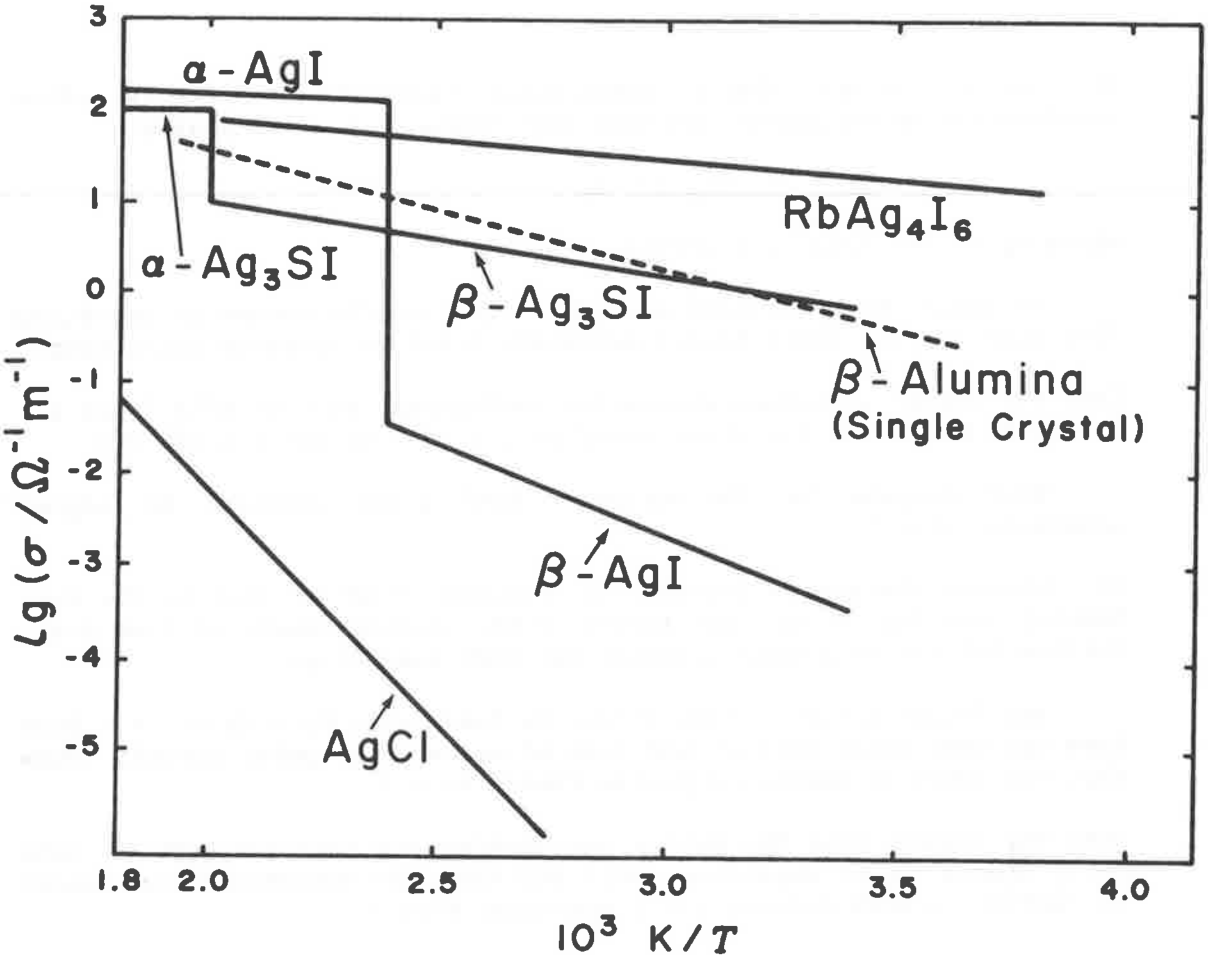
Essay Title: Rate processes on surfaces in the context of capture and diffusion of adsorbed species.

Questions:

H1 Write brief notes on *three* of the following:

(a) defects in solids such as ZnO, NiO, FeO, and their role in determining the electrical conductivity and catalytic activity;
(b) interaction between point defects and the formation of clusters;
(c) the density of states for amorphous silicon. Comment on the doping of amorphous silicon required for electronic devices;
(d) reconstruction of surfaces and surface states;
(e) activation volumes, and a method used for their determination.

H2 What information can be obtained from measurements of the electrical conductivity of ionic solids as a function of temperature?



The figure gives published data of the electrical conductivity of a number of solids. Explain the form of the results.

Comment briefly on the use of fast-ion conductors in high energy-density batteries.

Group I HIGH ENERGY ASTROPHYSICS (Minor Option)

Essay Title: The evidence for, and physics of, accretion onto neutron stars.

Questions:

I1 Derive, using simple approximate arguments, the mass-radius relation for white dwarfs and show that there is a maximum mass,

$$M_{\mathrm{Ch}} = (\hbar c/Gm_{\mathrm{p}}^{2})^{3/2} m_{\mathrm{p}}$$

where m_{p} is the mass of a proton.

At about what luminosity will a spherically-symmetric accretion flow onto a white dwarf become optically thick to electron scattering?

[You may assume a Thomson absorption coefficient of 0.04 $\mathrm{m^2 kg^{-1}}$ and a mass and radius for the white dwarf of 2×10^{30} kg and 5×10^{6} m.]

What happens to the emergent hard X-ray spectrum at higher accretion rates?

I2 Discuss the energy sources for a pulsar (such as that in the Crab Nebula) and for a pulsing binary X-ray source (such as Cen X-3). Outline the observational evidence for your reasoning.

Two bright X-ray sources differ in luminosity by a factor L. Both have the same pulse period, and rate of decrease of pulse period. Show that the ratio of surface magnetic fields is L^{-3}.

[You may assume that the matter surrounding the magnetosphere of each X-ray source is in Keplerian orbit and that the magnetospheric radius is similar to that deduced for a spherical flow.]

Group J LASERS AND MASERS (Minor Option)

Essay Title: Kilohertz and picoseconds in laser physics.

Questions:

J1 Discuss the significance of *saturation* in laser physics. Your answer should include reference to:

(a) Einstein theory of spontaneous and stimulated emission;
(b) laser linewidth;
(c) picosecond pulses;
(d) spectroscopy;
(e) maser amplification.

J2 Compare the operations of pulsed ruby and nitrogen lasers, drawing attention to the similarities and differences.

Explain also the following observations:

(a) The output of a ruby laser often comprises a ragged series of pulses. A single pulse, of length about 10 ns, can be obtained by including a cell of nitrobenzene in the laser cavity.

(b) The output of a nitrogen laser is usually a single clean pulse of duration about 1 ns, and may have a divergence of 0.05 radian.

Group K SCIENCE, TECHNOLOGY AND SOCIETY (Minor Option)

Essay Title: Scientific knowledge and public choice in energy and the environment.

[Each candidate is expected to choose a special topic within this area and to indicate his or her choice by a sub-title.]

There are no questions in this Group.

Group L MACROMOLECULAR PHYSICS (Minor Option)

Essay Title: Miscibility of polymers.

Questions:

L1 Outline the Flory argument that for a long linear polymer chain in a good solvent

$$R = aN^{\nu}$$

where $\nu = 3/5$, N is the number of units in the chain, R is a measure of the radius of the coil, and a is a material constant. Comment on the treatment of fluctuations.

If a sufficiently small force of magnitude f, is applied across the ends of the chain, show that their mean separation r obeys

$$r/R \propto fR/kT$$

and give a reasoned argument for the form of the relation between r and f for larger forces. Explain the idea of a 'blob', and interpret your results in those terms.

[TURN OVER for continuation of Question L1

A polymer solution is crosslinked such that there is typically a large number, N_1, of monomers between crosslink points. At what concentration would you expect it to be in osmotic equilibrium with pure good solvent? Estimate the elastic modulus of such a gel and, from the single chain results above, the variation with λ of tensile force $F(\lambda)$ required to stretch a gel sample by a *large* ratio λ in one direction. You may ignore entanglement effects in your answer. If the stretched state is allowed to restore osmotic equilibrium, what happens to its transverse dimensions?

L2 Outline the derivation of the Rouse equation for the dynamics of a linear chain in dilute solution:

$$\partial \mathbf{r}/\partial t = \epsilon\ \partial^2 \mathbf{r}/\partial n^2$$

and find its normal modes.

What effects in real polymers are ignored in this treatment?

To what extent can this model be applied to chains in a polymer melt?

Show how the reptation model leads to the predictions for the viscosity, η, and diffusion coefficient D:

$$\eta \propto M^3, \qquad D \propto M^{-2},$$

for sufficiently large relative molecular mass M. To what extent are these laws observed in practice?

Group M PLASMA PHYSICS (Minor Option)

Essay Title: The plasma physics of laser fusion.

Questions:

M1 Comment on the main features of the behaviour of charged particles in magnetic fields and describe how drift motions arise in the presence of a magnetic field and an additional force.

Describe the magnetic field configuration in a Tokamak, emphasizing the role played by each component of the field in confining the plasma.

~~M8~~ The solar wind in the vicinity of the Earth is a hydrogen plasma with number density $8\times10^{6}\mathrm{m}^{-3}$, ion temperature $T_i \approx 5\times10^{4}$K, electron temperature $T_e \approx 5T_i$ and with a magnetic field strength of about 5×10^{-9} T. It streams past the Earth at a speed of 400 km s^{-1}. The Earth's magnetic field is 3×10^{-5} T on the equator and drops as r^{-3} with increasing distance r from the centre of the Earth. Give approximate expressions for the following quantities and comment briefly on their physical significance:

(a) the plasma frequency;
(b) the Debye length and number;
?(c) the collision frequency and mean free path;
(d) the Alfvén speed;
(e) the sound speed;
(f) the position of the shock which forms around the Earth.

Group N PURE AND APPLIED SEISMOLOGY (Minor Option)

Essay Title: Determining the Earth's density distribution using seismology.

Questions:

N1 Define the ***Bullen ray parameter*** and derive the formula for the range of a seismic ray in a horizontally stratified medium. Show how velocity-depth curves may be obtained from travel-time curves. Explain why we cannot determine the seismic velocity in and below a low velocity zone using a travel-time curve. How might this difficulty be overcome?

[Note $\int(a^2-p^2)^{-½}(p^2-b^2)^{-½}p\,dp = -\tan^{-1}[(a^2-p^2)/(p^2-b^2)]^{½}$.]

N2 A reflection seismic dataset is stacked and assumed to be equivalent to a zero-offset section. In order to explain and interpret the observed data, define the following terms:

zero-offset section; record surface; reflector surface; migrator's equation; point diffractor; surface of maximum convexity; exploding reflector model; v/2 model.

Describe the geometrical procedure for migration to correct the apparent position of a reflector to the true position. Assume the medium above the reflector is homogeneous. Outline how the method can be extended to media where the velocity varies with depth.

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Group O CONTINUUM MECHANICS (Minor Option)

Essay Title: Waves in continuous media.

Questions:

O1 Give a brief statement of the reasons for instabilities in continuous systems. Illustrate your answer by an outline discussion of the stability of Poiseuille flow.

Estimate the transition velocity between laminar and turbulent flow for a channel 200 mm x 10 mm.

[Kinematic viscosity of water $\approx 1\ mm^2s^{-1}$.]

O2 Give an account of the general procedures to be followed to obtain the mean transport coefficients or moduli of continuous media containing a dilute distribution of inclusions. Illustrate your answer by deriving the viscosity of a suspension of spheres in Stokes flow.

Group P MEDICAL PHYSICS (Minor Option)

Essay Title: Factors influencing the choice of measuring devices for ionizing radiation, when used for medical applications.

Questions:

P1 Explain briefly why protons are particularly suitable for nuclear-magnetic-resonance (NMR) imaging.

A patient is arranged horizontally in a static vertical magnetic field that has a mean value of 0.15 T but varies at the rate of $10\ mT\ m^{-1}$ along the long axis of the patient. Suggest a suitable pulse profile to activate the protons in a vertical slice 15 mm thick.

Discuss the possible sources of electrical and electromagnetic hazard in NMR imaging.

[Gyromagnetic ratio of the proton = $2.68 \times 10^8\ rad.s^{-1}T^{-1}$.]

P2 Explain the terms *objective contrast* and *subjective contrast* in connection with medical imaging and suggest reasons why they may be different.

Discuss the main factors which determine and influence objective contrast in:

(a) a conventional diagnostic radiograph using film;
(b) a nuclear medicine image acquired on a computer and displayed on a monochrome video display.