Primordial Magnetic Fields and the Large-Scale Structure of the Universe

It is proposed that a large-scale primordial magnetic field with a present strength of $\sim 10^{-9}$ Gauss which is tangled on scale ~ 1 Mpc could have significant effects on the formation of large-scale structure. Such a field is consistent with observational constraints, primordial nucleosynthesis and with the existence of strong (micro-Gauss) magnetic fields in galactic proto-disks at high redshift. Although there is no compelling physical mechanism for its generation, a primordial magnetic field of this magnitude could provide the extra fluctuations needed to reconcile theories of galaxy formation with observations of large-scale structure.

Key Words: *cosmology, galaxy formation, magnetic fields*

1. INTRODUCTION

Observational studies of galaxy clustering are placing a great deal of pressure on the standard gravitational instability picture of galaxy and cluster formation. Quantitative measures of clustering obtained from the APM survey[1] and the QDOT IRAS galaxy redshift survey[2,3] are in clear disagreement with the predictions of the standard version of the Cold Dark Matter model.[4,5] In the CDM picture, large-scale structures form hierarchically by the progressive merging of smaller sub-units. Although the theory is remarkably successful on the scales of galaxies, it fails to predict the correct level of clustering on very large scales. If the theory is

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normalised to produce the correct amount of large-scale clustering then the agreement of the model with galaxy properties is lost and the predicted level of anisotropy in the microwave background radiation is increased.[6] Attempts to increase the large-scale clustering by a straightforward change in the shape of the initial power spectrum are also severely constrained by microwave background anisotropy measurements.[7]

Since the standard CDM model incorporates the assumption that the density fluctuations were gaussian, one way to reconcile CDM with the observations would be to invoke some form of non-gaussian primordial fluctuations which could allow one to alter the normalisation of the power spectrum on large scales while, at the same time, still forming objects satisfactorily on scales up to those of galaxies. One possibility is the cosmic texture scenario where three-dimensional topological defects induce a non-gaussian distortion on interesting scales.[8,9] Another way is to invoke a more complicated scale-dependent bias in the ratio of dark matter to luminous galaxies than has previously been contemplated.[10]

The problem with Hot Dark Matter (HDM) models has been of the opposite nature. In such models, large-scale structure forms first because of the collisionless damping of small-scale density fluctuations. Here the problem is that one has to make superclusters first; galaxies then form by fragmenting out of gas cooling in the pancakes formed by anisotropic collapse on supercluster scales. In this picture one generates too much large-scale clustering.[11,12] A recent suggestion on how to avoid this pitfall was made by Villumsen, Scherrer and Bertschinger[13] who invoke population of point-like seeds, such as primordial black holes or non-topological solitons, that can generate galaxy-scale objects before the onset of pancake collapse. The remedy here is rather similar to that in the CDM picture in that it involves the existence of a set of non-gaussian perturbations on galaxy scales. It is even possible that one could use such a mechanism to reconcile baryon-only models with observations without violating microwave background constraints.[14,15]

Given license to invoke an arbitrary non-gaussian density distribution (or, equivalently, an arbitrary scale-dependent bias factor) one could no doubt reconcile almost any initial fluctuation spectrum with observations. The mechanisms so far proposed are based either on highly speculative physics (cosmic textures, primordial black holes) or complex astrophysical processes whose influence is unlikely to be amenable to direct test for some time.[10]

It is the purpose of this paper to demonstrate that magnitude fields could provide a physical mechanism capable of generating a higher level of large-scale clustering than standard models. Drawing heavily on the pioneering paper of Wasserman[16] we shall show that a primordial relic magnetic field with present-day strength of order 10^{-9} which is tangled on the scale of a few Mpc* would have been dynamically important at $z = 1000$ and could therefore have seeded galaxy formation directly regardless of any primordial density fluctuations. This idea is certainly not new. It was first discussed by Zel'dovich[17] and Rees and Reinhardt,[18] then refined by Wasserman[16] and Peebles.[19] The motivation for this paper is to stress that, even in models where non-baryonic (and presumably uncharged) dark matter dominates, magnetic fields can still be crucial in determining the resulting large-scale structure if they possess the right strength and spatial distribution. The general implications of large-scale magnetic fields were discussed by Rees[20] who concentrated on the possible effects on cooling and star formation and the bias this might produce in the luminous matter distribution; even a very weak field may produce a large scale-dependent bias of the type actually required by standard CDM. Here we shall go one step further and argue that magnetic stresses may have been large enough to form density perturbations directly at high redshift and that the required field is not only consistent with observational limits but is interesting close to the value needed to explain the field observed in galaxies,[21] high redshift absorption systems[22,23] and rich cluster environments.[24,25]

2. GROWTH OF DENSITY PERTURBATIONS IN THE PRESENCE OF A MAGNETIC FIELD

In this section we use an argument originally due to Wasserman[16] to show that a large-scale magnetic field can act as a source for

*Where necessary we shall use $H_0 = 100h$ kms^{-1} Mpc^{-1} to compute distances.

density perturbations capable of forming galaxies and large-scale structures by the present epoch. Initally we assume that all the matter couples to the magnetic field. At the end of this section we shall consider how the argument is altered if there exists dark matter which does not feel the electromagnetic force. For general references on magnetic fields see Parker[26] and Rees.[20]

To study the growth of density perturbations in a universe pervaded by a magnetic field we have to add a magnetohydrodynamic term to the usual Newtonian force equation:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \left(\frac{\dot{a}}{a}\right) \mathbf{v} + \left(\frac{\mathbf{v} \cdot \nabla}{a}\right) \mathbf{v} = -\rho \frac{\nabla \Phi}{a} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi a}. \quad (1)$$

In this equation and throughout this paper $a(t)$ is the cosmological scale factor, Φ is the peculiar gravitational potential, $\mathbf{B}$ is the magnetic field (in gauss), ρ and $\mathbf{v}$ are the density and velocity fields, respectively, and the equation is written using comoving coordinates. The role of matter pressure gradients is neglected. The equation of continuity for the matter fluid is

$$\frac{\partial \rho}{\partial t} + 3\left(\frac{\dot{a}}{a}\right) + \frac{\nabla \cdot (\rho \mathbf{v})}{a} = 0, \quad (2)$$

and we also have Poisson's equation:

$$\nabla^2 \Phi = 4\pi G a^2 (\rho - \rho_0) \quad (3)$$

where ρ_0 is the density of the homogeneous background model. To these equations we add Maxwell's equations in comoving coordinates:

$$\nabla \cdot \mathbf{B} = 0 \quad (4)$$

and, expressing conservation of flux,

$$\frac{\partial}{\partial t} (\mathbf{B} a^2) = \frac{\nabla \times \mathbf{v} \times a^2 \mathbf{B}}{a} \quad (5)$$

where we have assumed that the electrical resistivity of the plasma is negligible: despite recombination the residual ionisation of the plasma is sufficient to keep it a good conductor and prevent ohmic dissipation of the field.[27]

The earliest time we can contemplate the field having an effect on density inhomogeneities is at $t = t_r$ when the plasma recombines and baryons are no longer prevented from moving by viscosity and radiation pressure. Let the magnetic field at this time be $\mathbf{B}_r(\mathbf{x}) = \mathbf{B}(\mathbf{x}, t_r)$. In subsequent sections we shall discuss where the magnetic field might have been generated and constraints on its properties, but for the moment we shall just take its existence as given. If flux is conserved,

$$\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_r(\mathbf{x}) a^2(t_r)/a^2(t) \tag{6}$$

by Eq. (5) (to zeroth order in perturbation theory). Defining a dimensionless density contrast,

$$\delta(t) = \frac{\rho_0(t) - \rho_0(t)}{\rho_0(t)}, \tag{7}$$

and linearising Eqs. (1) and (2) in $\mathbf{v}$ and δ leads to the following equations:

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\frac{\dot{a}}{a}\right)\mathbf{v} = -\frac{\nabla\Phi}{a(t)} + \frac{[(\nabla \times \mathbf{B}_r) \times \mathbf{B}_r]a^4(t_r)}{4\pi\rho_0(t)a^5(t)} \tag{8}$$

and

$$\frac{\partial \delta}{\partial t} + \frac{\nabla \cdot \mathbf{v}}{a(t)} = 0. \tag{9}$$

Taking the divergence of (8) and using (3) and (9) leads to

$$\frac{\partial^2 \delta}{\partial t^2} + 2\left(\frac{\dot{a}}{a}\right)\frac{\partial \delta}{\partial t} - 4\pi G\rho_0(t)\delta = \frac{\nabla \cdot [(\nabla \times \mathbf{B}_r) \times \mathbf{B}_r]a^4(t_r)}{4\pi\rho_0(t)a^6(t)}. \tag{10}$$

At redshifts $1 + z > 1/\Omega_0$ it is a good approximation to set $\rho_0(t) = 1/6\pi G t^2$ and $a(t)/a(t_r) = (t/t_r)^{2/3}$. We also know that $\rho_0(t)a^3(t)$ must be constant by the zeroth-order solution to (2). These considerations lead to the following equation for δ:

$$\frac{\partial^2\delta}{\partial t^2} + \frac{4}{3t}\frac{\partial\delta}{\partial t} - \frac{2\delta}{3t^2} = \left(\frac{t_r}{t}\right)^2 \frac{\nabla\cdot[(\nabla \times \mathbf{B}_r) \times \mathbf{B}_r]}{4\pi\rho_0(t_r)a^2(t_r)}. \qquad (11)$$

To estimate order-of-magnitude effects we assume that the magnetic field is random with some rms value B_r and (coordinate) coherence length λ_B. The term on the right-hand side of (11) can be written as $\sim B_r^2/\lambda_B^2$. Obviously the exact value of this term varies in a complicated way on the detailed configuration of the field. A statistically homogeneous random-phase field, where each component of **B** is gaussian and random, would seem to offer favourable sites for proto-objects at peaks of the field given by $\nabla\cdot[(\nabla \times \mathbf{B}) \times \mathbf{B}]$ rather than peaks of the initial density field δ. The statistics of a random gaussian magnetic field are rather more difficult to compute than a field such as the linear velocity field since the requirement that $\nabla\cdot\mathbf{B} = 0$ means that different components of **B** are not independent. The details of the non-gaussian imprint **B** leaves in the density field will be left to future numerical work; the statistics will clearly be non-gaussian. We write

$$\frac{\partial^2\delta}{\partial t^2} + \left(\frac{4}{3t}\right)\frac{\partial\delta}{\partial t} - \frac{2\delta}{3t^2} = \left(\frac{t_r}{t}\right)^2 \frac{B_r^2}{4\pi\rho_0(t_r)a^2(t_r)\lambda_B^2}. \qquad (12)$$

The solution to this second-order inhomogeneous equation can be found by constructing a Green's function from the homogeneous equation with the boundary conditions $\delta(t_r) = (\partial\delta/\partial t)_{t_r} = 0$. The solution is[16]

$$\delta(t) = \frac{t_t^2 B_r^2}{4\pi\rho_0(t_r)a^2(t_r)\lambda_B^2}\left[\frac{9}{10}\left(\frac{t}{t_r}\right)^{2/3} + \frac{3}{5}\left(\frac{t_r}{t}\right) - \frac{3}{2}\right]. \qquad (13)$$

The upshot of this calculation is that the magnetic field stresses have produced a δ which grows at late times like $t^{2/3}$ so that even if there were no actual density inhomogeneities present at $t = t_r$

the density field at late times will look as if there were an initial δ_r at $t = t_r$ of order

$$\delta_r \simeq \frac{B_r^2 t_r^2}{4\pi\rho(t_r)a^2(t_r)\lambda_B^2}. \tag{14}$$

To seed galaxies directly the effective δ_r must be of order 10^{-3} assuming a flat Universe. In an open Universe fluctuations begin to freeze out at $z = 1/\Omega_0 - 1$ so one requires an initial fluctuation about a factor $1/\Omega_0$ higher. An additional complication occurs if the universe is dominated by a form of dark matter that does not interact appreciably via the electromagnetic force (i.e., all weakly interacting non-baryonic dark matter candidates). To treat this case we really have to write down an extra equation describing the solely gravitational interaction of perturbations to the gas density upon the dark matter. However, since the magnetic pressure falls so quickly ($\propto a^{-4}$) one can see that to produce a gravitational effect on the total density field one would need to induce a δ_r in the gas roughly a factor Ω_0/Ω_b larger than the $\Omega_0 = \Omega_b$ case (assuming all the baryons are in gas at $z = 1000$). If $\Omega_0 = 1$ and $\Omega_b = 0.05$ then the required B-field is a factor ~ 4.5 larger. In all cases the perturbations are induced on a scale $\sim \lambda_B$; a value $\lambda_B \sim 1$ Mpc corresponds to a bright galaxy whereas a scale $\lambda_B \sim 10$ kpc corresponds to more like a globular cluster mass. Putting these considerations together we find that, to have a direct seeding effect on cosmic structure, we require

$$B_r \simeq 8 \times 10^{-3} \left(\frac{\lambda_B}{h^{-1}\ \mathrm{Mpc}}\right) (\Omega_b h^2)^{1/2}\ \mathrm{Gauss}. \tag{15}$$

The consequences of the perturbations seeded by such a field will be different depending on the form of the dark matter. In a Hot Dark Matter (HDM) model there are essentially no perturbations at $t = t_r$ on scales less than superclusters, so the B-field will act to seed fluctuations which can collapse to form galaxies before pancake collapse produces the large-scale structure, rather like the seeded HDM models of Villumsen, Scherrer and Bertschinger.[13] However, there is a complication in this picture due to the fact that at $z = 1000$ the neutrinos that make up the dark matter still exert considerable pressure so that the neutrino Jeans mass is larger than a galaxy scale. Although magnetic stresses can move the baryons around, the neutrinos will not begin to cluster until the Jeans mass falls to the correct scale which occurs around $z \sim 150$ ($M_J \propto (1 + z)^{3/2}$). The initial perturbation must therefore be a factor $\sim 1000/150$ larger and the magnetic field a corresponding factor of 3 or so larger. In a CDM model, however, there will be initial density fluctuations on scales down to 10 kpc. Here the competition between magnetic forces and gravitation-driven hierarchical clustering is much more complicated. It seems likely, however, that the effect will be to introduce a non-gaussian pattern of perturbations at a scale $\sim \lambda_B$ rather like the currently fashionable cosmic textures.[8,9] The magnetic field postulated here may also allow one to reconcile baryon-only models with the constraints provided by the observed isotropy of the microwave background temperature on the sky. In the usual picture structures are formed from density perturbations that are generated prior to recombination and are therefore accompanied by radiation brightness fluctuations which must be large if the perturbations are large enough to form galaxies and clusters by the present time.[14,15] In the magnetic field picture there need be no density fluctuations at all until after recombination at which point they are induced by the magnetic field stresses.

Incidentally, one other artifact of a primordial magnetic field of this magnitude would be the generation of large-scale vorticity in the galaxy peculiar velocity field and possibly even the origin of the angular momentum of galaxies.[16] However, since the vorticity decays as a^{-1} while the curl-free velocity generated by the gravitational potential grows at least as a^{+1}, it is unlikely that this will have any observational consequences at the present time.

3. OBSERVATIONS OF, AND CONSTRAINTS ON, LARGE-SCALE *B*-FIELDS

Equation (15) shows the minimum B-field required at $t = t_r$ in order that magnetic stresses should induce density perturbations large enough to form non-linear objects by the present time. If we let such a field expand with the Universe conserving flux then $B \sim a^{-2}$. The minimum relic field in intergalactic space now should therefore obey:

$$B_0 \simeq 8 \times 10^{-9} \left(\frac{\lambda_B}{h^{-1}\,\mathrm{Mpc}}\right) (\Omega_b h^2)^{1/2} \text{ Gauss.} \qquad (16)$$

This assumes that the field has undergone a free expansion. In reality the field is embedded inside cosmic structures which collapse to form dense objects. Since the dynamical effect of the B-field decays as a^{-4} while the density perturbations grow as a^{+1} in linear gravitational instability, a collapsing structure will simply squeeze the field and amplify it by a factor $(\rho_{obj}/\rho_g)^{2/3}$ where ρ_{obj} and ρ_g are the gas densities inside the structure and in the background respectively. Using this argument it is not difficult to see how to generate a galactic B-field by simple compression.[20,21] A compression of ρ by $\sim 10^4$ leads to a B_{gal} of ~ 4 μG using (16). One can similarly imagine how to generate the B-fields observed by Faraday rotation techniques by rich cluster environments such as Abell 2319[24] and Cygnus A[25]; see also the book by Kolb and Turner[28] and references therein.

The traditional explanation for galactic magnetic fields is the operation of some form of dynamo.[21,26] Although these do require a seed field it may be very weak and could be formed, for example, in the first generation of stars. However, recent studies of damped Lyman-α absorption systems in QSO spectra show that these too have strong Faraday rotation indicating the presence of magnetic fields up to tens of micro-Gauss.[22,23] If the usual interpretation of these objects (that they represent the early stages of galactic disk formation) is correct, then it is hard to see how a dynamo can have had time to wind up the field to a significant extent.

The qualitative picture therefore looks rather plausible. Let us try to constrain the possible values of B_0 and λ_B using observations. We discuss below the three strongest constraints on these parameters. Others are discussed, for example, by Rees and Reinhardt[18] and Rees.[20] Model-dependent constraints deriving from the isotropy of the cosmic microwave background are discussed by Madsen.[29]

Intergalactic Faraday Rotation

We discuss above how Faraday rotation measurements imply a B-field up to tens of micro-Gauss in strength. These measurements do not test the true primordial field since they are performed in dense environments where the field will have been compressed. An upper limit on the Faraday rotation of background sources leads to the following constraint[17,20] on the intergalactic relic **B**:

$$B_0 < 1.6 \times 10^{-7} \left(\frac{\lambda_B}{1\ \text{Mpc}}\right)^{-1/2} (\Omega_g)^{-1}\ \text{Gauss.} \qquad (17)$$

Here the present gas density contributes a fraction Ω_g to the critical density but that may be rather less than the total baryon density since a substantial fraction of Ω_b may be in the form of stars or low-mass objects. Note that, although the field may have been trapped in proto-objects at high redshift, there will still be an intergalactic field at the present time either because some objects eject their field when they merge or because only those regions with strong field *gradients* (11) ever trap the field in the first place. However, the result (17) is far from providing a strong constraint on the requirement (16), even if Ω_b is very low.

Jeans Length

Recall that in Section 2 we neglected the term in the force equation (1) deriving from gradients in the gas pressure. It is this term that leads to the existence of a minimum Jeans length, λ_J, below which density perturbations are prevented from collapsing by gas pressure: $\lambda_J = c_s(\pi/G_p)^{1/2}$ where ρ is the gas pressure and c_s is the sound speed. This turns out to be roughly

$$\lambda_J \simeq 0.25 \left(\frac{T}{3000\ \text{K}}\right)^{1/2} (\Omega_b h^2)^{-1/2}\ \text{Mpc} \qquad (18)$$

for a hydrogen plasma at temperature T, again assuming that all the baryons are in the form of gas at $z \simeq 1000$. Close to recombination one expects the residual ionisation of the gas to maintain T close to the radiation temperature but at later times the gas will cool adiabatically with $T \sim a^{-2}$. If we are to invoke a magnetic field to form structures we have to ensure that the magnetic pressure dominates the gas pressure. The easiest way to parameterise this is to note that the Afvén speed is $c_a^2 \simeq B^2/8\pi\rho$ so that there exists a magnetic Jeans length λ_{BJ} given by

$$\lambda_{BJ} \simeq c_a \left(\frac{\pi}{G\rho}\right)^{1/2} = \frac{B_r}{\rho_0(t_r)\sqrt{8G}} \tag{19}$$

at time $t = t_r$. Equation (14) can now be re-written as

$$\delta_r \simeq \frac{2\rho_0(t_r)Gt_r^2}{\pi}\left(\frac{\lambda_{BJ}}{\lambda_B}\right)^2 = \frac{1}{3\pi^2}\left(\frac{\lambda_{BJ}}{\lambda_B}\right)^2, \tag{20}$$

using $6\pi G\rho_0(t)t^2 = 1$. The requirement that $\delta_r \simeq 10^{-3}/\Omega_b$ can be succinctly stated[19] as $\lambda_{BJ} \simeq 0.17\Omega_b^{-1/2}\lambda_B$. But in order that magnetic stresses should dominate the pressure, we require $\lambda_{BJ} > \lambda_J$. Using the above results (18) and (20), this becomes

$$\lambda_B > 1.5h^{-1}\ \mathrm{Mpc}. \tag{21}$$

The scale length cannot therefore be made arbitrarily small as one might think from (17). Interestingly, the magnetic Jeans mass corresponding to the scale (21) is close to that of a bright galaxy $\sim 10^{11}\ M_\odot$; the model therefore produces an alternative explanation for galaxy sizes independent of the cooling properties of gas at high redshift.[4]

Big–Bang Nucleosynthesis

Barrow[30] found that the fraction of Helium-4 produced in primordial nucleosynthesis falls outside observational limits if the expansion of the Universe is too fast or too slow compared to the case where the expansion is solely due to the energy density in thermal radiation. These considerations basically constrain the ratio of the energy density in the magnetic field to that in the radiation background to:

$$Q = \frac{B_0^2}{8\pi\rho_R} \lesssim \frac{2}{3}. \tag{22}$$

Note that the flux-conserving expansion (6) keeps the ratio Q constant since B^2 and ρ_R both vary as a^{-4}. This gives a limit of

$$B \leq 3 \times 10^{-6} \text{ Gauss.} \qquad (23)$$

Obviously this assumes that the B-field exists at the time of nucleosynthesis: it could actually have been generated by plasma processes occurring at rather late times, as we discuss below.

4. ORIGINS?

So far we have postulated the existence of a magnetic field with the properties required to influence the onset of galaxy formation. We must now speculate on the possible physical origin of such a field. Many mechanisms have been suggested that might generate a large-scale magnetic field[20,28] but many of these involve non-linear structure formed after recombination. For the idea outlined in this paper to work we need physical processes to generate the field before recombination.

Harrison[31] suggests a simple mechanism for generating a large-scale B through vortical perturbations to the matter fluid. Photons in the thermal radiation are strongly coupled to electrons at $t < t_r$ by Thomson scattering, but are less strongly coupled to the ions. While the energy-density of the photon–electron component of the plasma exceeds its rest mass energy density, it behaves as a single fluid for which the energy density falls off as $\rho \sim a^{-4}$. Conservation of angular momentum in this fluid requires that the angular velocity of a turbulent eddy decays as a^{-1}. Since the rest mass of the ions dominates, their energy density falls only as $\sim a^{-1}$ and eddies in this component decay as a^{-2}. The difference in slow-down rates generates a current and consequently a magnetic field. However, the back-emf generated ensures that the slow-down rates couple together and the generation of a magnetic field by this process is consequently slow. Harrison estimates the maximum field strength that can be generated by such a mechanism is $\sim 10^{-18}$ Gauss which, although perhaps enough to feed a galactic dynamo, is too small to be of interest here.

The general idea that the primordial plasma could have been highly turbulent is now rather unfashionable and it is true to say that the lack of any perceptible spectral distortions in the cosmic microwave background places strong constraints on any turbulent dissipation of energy at high redshifts.[32] In fact we can place a limit on the primordial B that could be dissipated at high redshifts using the COBE spectral distortion measurements. If the time-scale for dissipation is short compared to the expansion time then we get $B < 2.4 \times 10^{-7}$ Gauss, a rather stronger limit than the nucleosynthesis constraint (23). It is not inconceivable that non-linear phenomena such as cosmic string-induced shocks or wakes or a Kolmogorov cascade operating in a turbulent primordial plasma[33–36] could be accompanied by the generation of a magnetic field of the correct order and scale length, without violating this constraint. It must be admitted, however, that it is very difficult to imagine a turbulent mechanism that can generate a sizeable magnetic field while leaving the matter distribution smooth enough to escape the constraints discussed above.

In the light of this discussion, it seems much more likely that the field we need would have to be generated in the very early Universe rather than in the period immediately prior to recombination. One such mechanism has been proposed by Turner and Widrow.[37] In this picture disordered magnetic fields are generated during an epoch of cosmic inflation by spontaneous symmetry-breaking involving the electromagnetic field. The fields produced are vanishingly small unless the unified model breaks the conformal invariance of the electromagnetic field. Possible models might involve gravitational couplings of the photon field; coupling the photon to a charged, massless nonconformally invariant scalar field or coupling the photon to an axion field. None of these models is particularly well-motivated and the results are very model-dependent, but the general idea certainly deserves further attention.

5. DISCUSSION

We have shown that a comparatively weak tangled primordial magnetic field of order 10^{-9} Gauss at the present time with a coherence length of up to a few Mpc could have generated sizeable density fluctuations at recombination and could therefore have acted as a direct seed for cosmic structures. If such a primordial field exists then it must have been dynamically important at $t = t_r$. The current observational constraints certainly do not rule out the existence of a primordial field with these properties. This suggests that magnetic fields could be the extra ingredient required to rescue flagging theories of galaxy formation whether of the hierarchical or pancake type. Of course, we have taken a rather extreme view: that magnetic fields could be the dominant dynamical factor and initial density perturbations may be small enough to be unimportant. If, and this seems somewhat more likely, there are both magnetic stresses and intrinsic density perturbations, then the field may influence structure in more subtle ways, some of which are discussed by Dekel and Rees.[10] A particularly interesting possibility is that an inhomogeneous magnetic field could modulate galaxy formation in the CDM picture by giving the baryons in some regions a streaming velocity relative to the dark matter; in some places the baryons might be prevented from falling into potential wells and the formation of luminous galaxies may be inhibited. Perhaps such a biasing scheme could reconcile the CDM model with clustering observations without requiring such a large field as we have invoked here.

The arguments here are based on order-of-magnitude estimates of the density perturbations induced by magnetic fields of a given scale length. We have been unable to calculate the detailed statistics of these perturbations nor study exactly how clustering develops into the non-linear regime in such a scenario. The fact that the dynamical effect of a primordial B actually decreases compared to self-gravity as a^{-5} suggests that N-body experiments involving magnetohydrodynamics ought to be feasible and such experiments are well worth exploring. Since MHD processes affect the growth of structure only at early times and non-linear gravity only at late times, all one need do is replace the usual Zel'dovich approximation for the initial conditions by an approximation scheme for solving the MHD equation (1). A study of the statistical properties of the field defined by $\nabla \cdot [(\nabla \times \mathbf{B}) \times \mathbf{B}]$ would also allow one to determine the clustering properties of maxima of this field as an estimate of the clustering of galaxies in the linear regime. We shall address these problems in future work.

It might be admitted that there is no compelling physical mechanism that might have generated the required field in the early Universe, although an inflationary mechanism has been suggested[37] and we cannot exclude an origin due to turbulent processes in the primordial plasma. Until we know how it is generated physically we do not know how to prescribe the statistical properties of the B-field. To explore fully the range of possible models involving different B, λ_B and statistical distributions will be an arduous task. There is, however, a considerable amount of evidence that magnetic fields do actually exist. Furthermore there is considerable evidence that the fields we see in cosmic objects have roughly the form that this model would predict. This certainly makes the scenario outlined in this paper worth considering and gives it more empirical motivation than those models based on entirely speculative physics such as cosmic textures.

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