



Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Leaving Certificate Examination 2026

Mathematics

Paper 1

Higher Level

Friday 5 June Afternoon 2:00 - 4:30

300 marks

Examination Number

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Date of Birth

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For example, 3rd February  
2005 is entered as 03 02 05

Centre Stamp

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## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer questions as follows:

- **any five** questions from Section A – Concepts and Skills
- **any three** questions from Section B – Contexts and Applications.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **any five questions** from this section.

**Question 1****(30 marks)**

(a) Solve the following equation in  $a \in \mathbb{R}$ ,  $a \geq -7$ :

$$5 = 1 + \sqrt{a + 7}$$

(b) Rearrange the following to write  $p$  in terms of  $n$  and  $s$ , where  $n$ ,  $p$ , and  $s$  are positive real numbers:

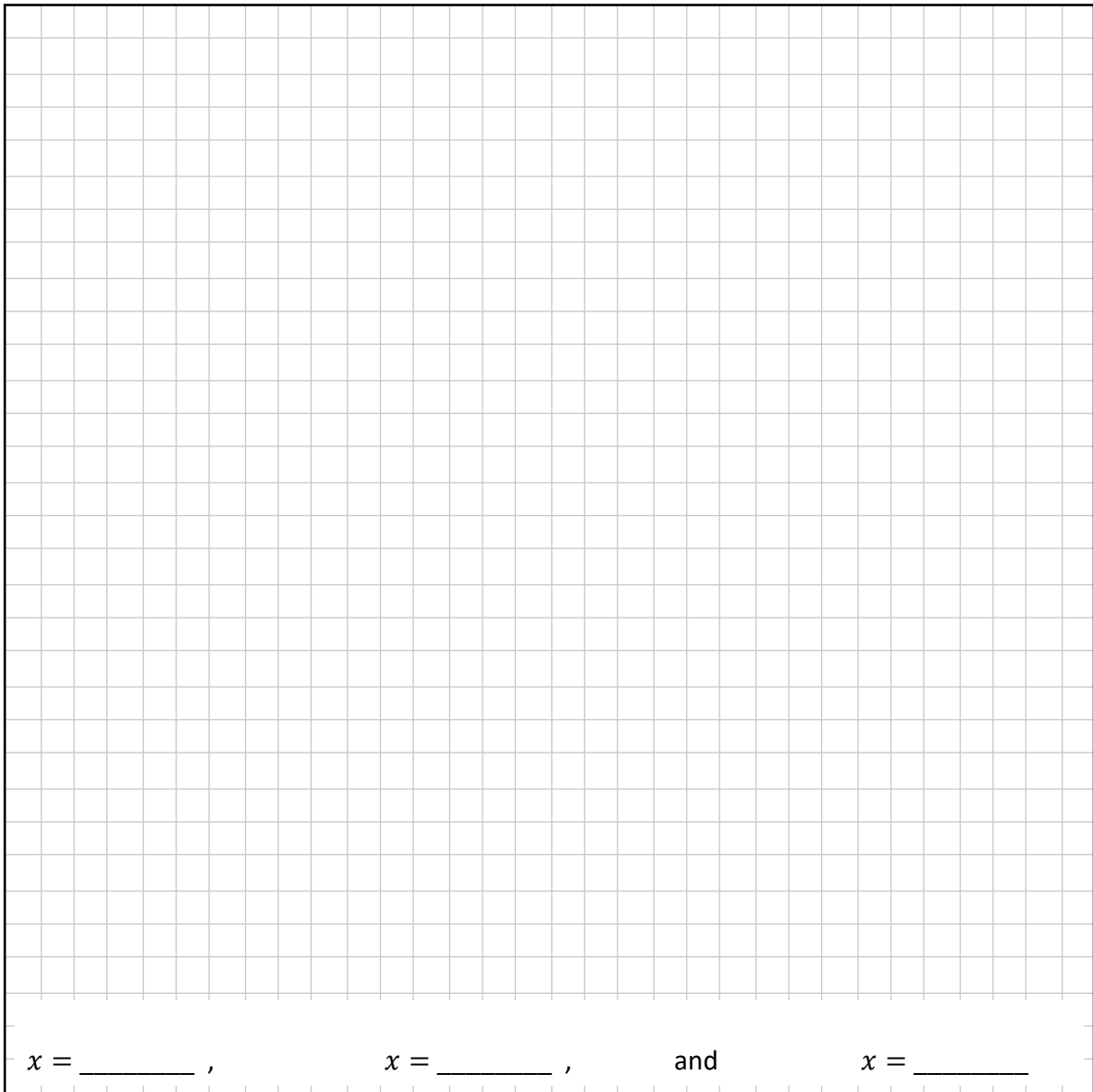
$$\frac{1}{n} + \frac{2}{p} = \frac{3}{s}$$

$p =$  \_\_\_\_\_

(c)  $2x^2 + 4x - 6$  is a factor of  $8x^3 + 6x^2 - 44x + 30$ .

Using this fact, or otherwise, solve the equation:

$$8x^3 + 6x^2 - 44x + 30 = 0$$



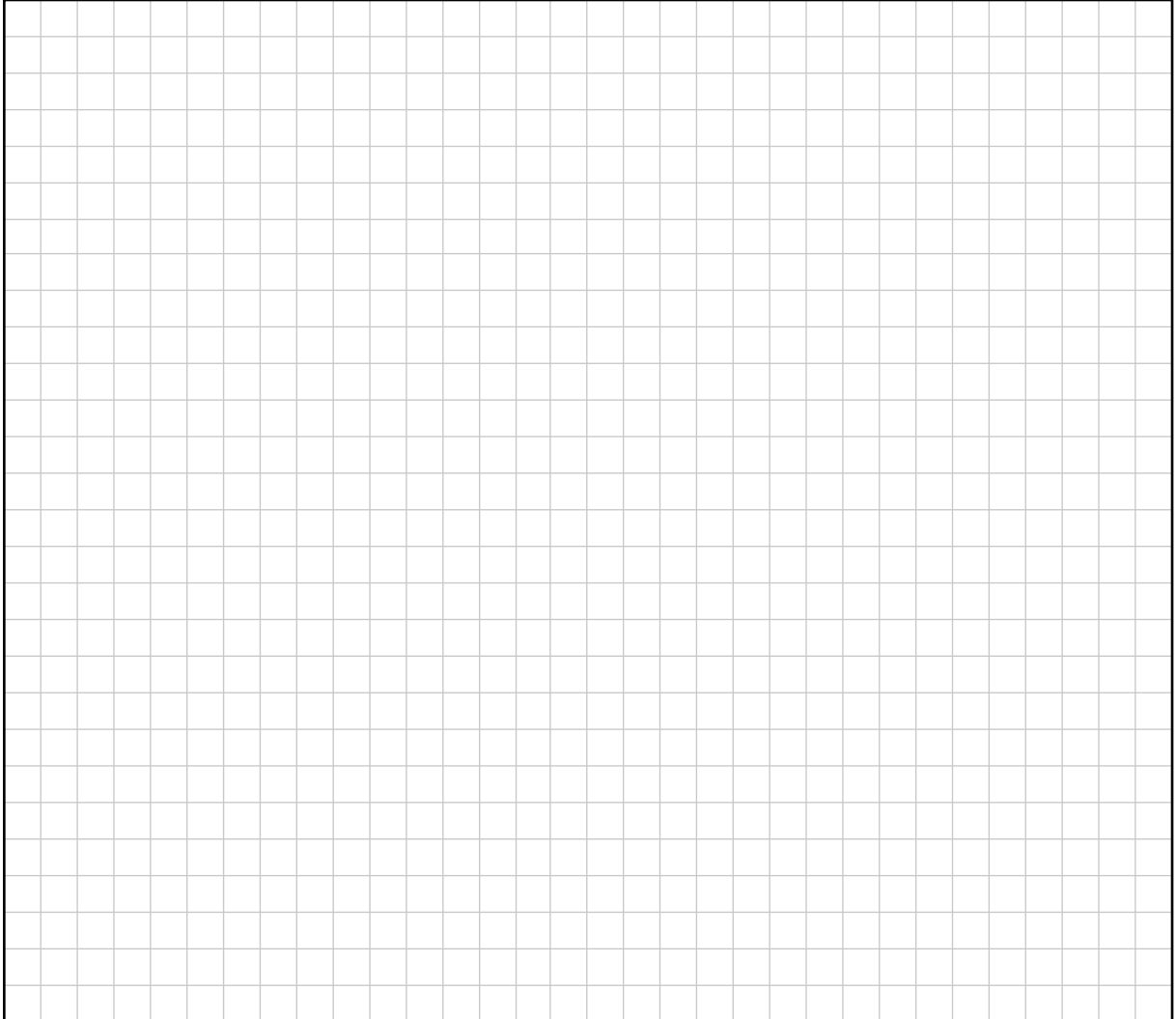
$x = \underline{\hspace{2cm}}$  ,                       $x = \underline{\hspace{2cm}}$  ,                      and                       $x = \underline{\hspace{2cm}}$

**Question 2**

**(30 marks)**

- (a) By writing the recurring part as an infinite geometric series, express the following number in the form  $\frac{a}{b}$  where  $a, b \in \mathbb{N}$ :

$$4.\dot{3}\dot{6} = 4.36363636 \dots$$



(b) Rya is taking out a loan to buy a car. The **annual** interest rate (APR) for her loan is 8.4%.

(i) Show that this APR is equivalent to a **monthly** interest rate of 0.67%, correct to 2 decimal places.

(ii) Rya's loan is for €14 000.

To pay off the loan, she makes equal repayments each month for 5 years. She makes the first repayment one month after she takes out the loan.

Find the amount of each repayment Rya makes, correct to the nearest cent, **and** hence find the total amount of **interest** that Rya pays over the 5 years.

Assume that the monthly interest rate for the loan is 0.67%.

Monthly repayment = € \_\_\_\_\_      Total interest = € \_\_\_\_\_



(b) The function  $g(x)$  has the following **second** derivative:

$$g''(x) = 30x - 18$$

The slope of the tangent to the curve  $y = g(x)$  at the point  $(-1, 8)$  is  $-2$ .

(i) By integrating  $g''(x)$ , use this information to find an expression in  $x$  for  $g'(x)$ , the **first** derivative of  $g(x)$ .

$g'(x) =$  \_\_\_\_\_

(ii) Hence, find an expression in  $x$  for  $g(x)$ .

$g(x) =$  \_\_\_\_\_

**Question 4**

**(30 marks)**

In this question,  $i^2 = -1$ .

- (a)  $u$  is the complex number  $u = 3 - 7i$ .  
 $\bar{u}$  is the complex conjugate of  $u$ .

Write  $2\bar{u} + 5iu$  in the form  $a + bi$ , where  $a, b \in \mathbb{R}$ .

- (b) Use **de Moivre's theorem** to find the value of  $(3 - \sqrt{3}i)^8$ .  
Give your answer in the form  $a + bi$ , where  $a, b \in \mathbb{R}$ .

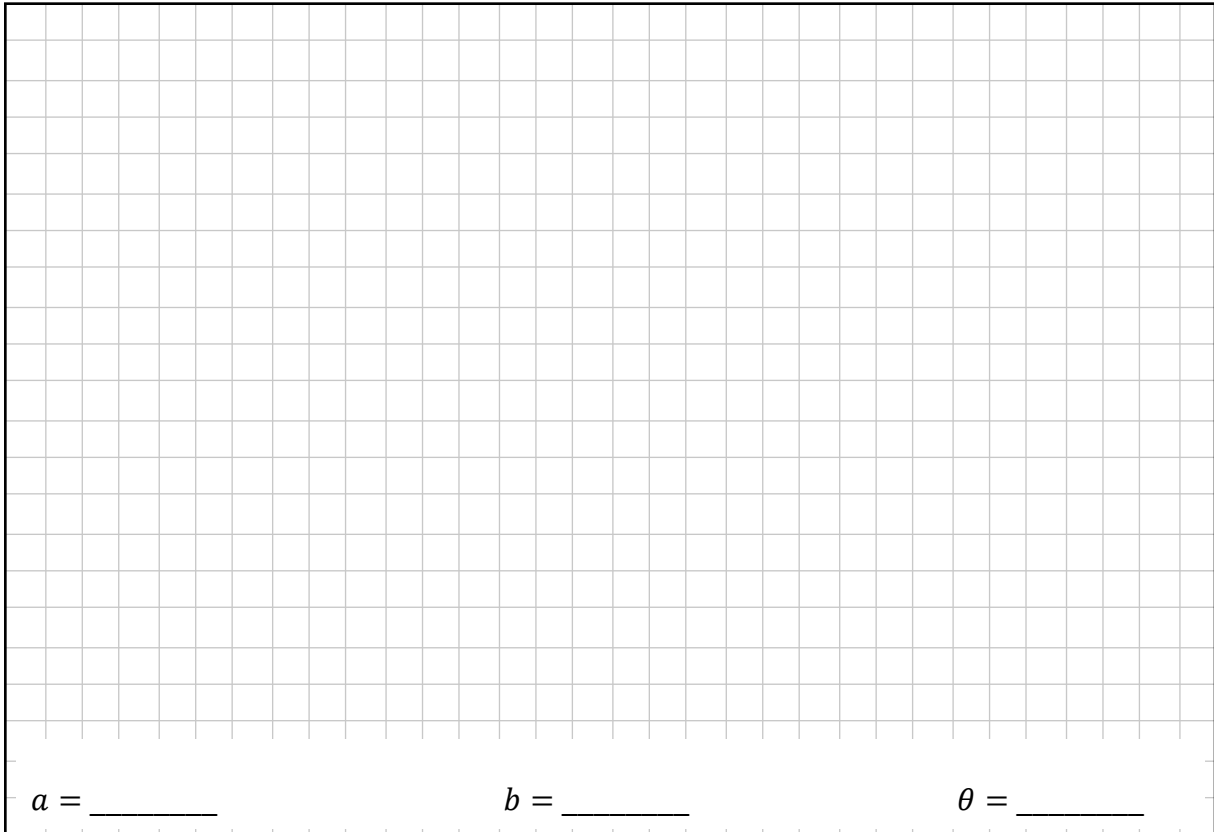
- (c)  $w$  and  $v$  are the following two complex numbers, where  $a, b \in \mathbb{R}$  and  $\theta$  is an angle in degrees:

$$w = a (\cos 160^\circ + i \sin 160^\circ)$$

$$v = b (\cos \theta + i \sin \theta)$$

For these two numbers,  $|w| = 5$  and  $vw = -15$ .

Use this information to find one possible set of values of  $a$ , of  $b$ , and of  $\theta$ .



$a =$  \_\_\_\_\_                       $b =$  \_\_\_\_\_                       $\theta =$  \_\_\_\_\_

**Question 5**

**(30 marks)**

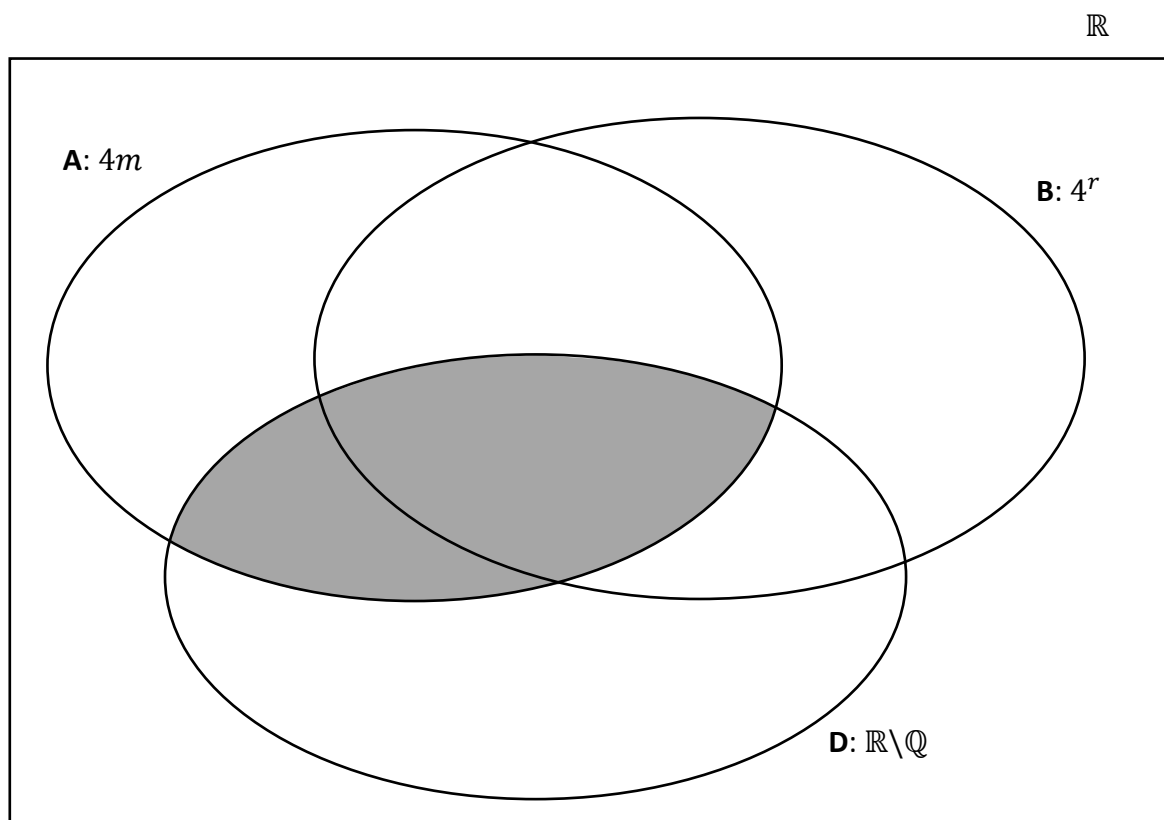
(a) Write each of the following numbers in the form  $4^r$ , where  $r \in \mathbb{Q}$ .

Number	64	$\frac{1}{16}$	2																																																																																																																																																																																																																																																																																																																																
Number in the form $4^r$	<table border="1"> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>																																																																																																					<table border="1"> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>																																																																																																															<table border="1"> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>																																																																																																														

(b) The Venn diagram below shows the following three sets in  $\mathbb{R}$ :

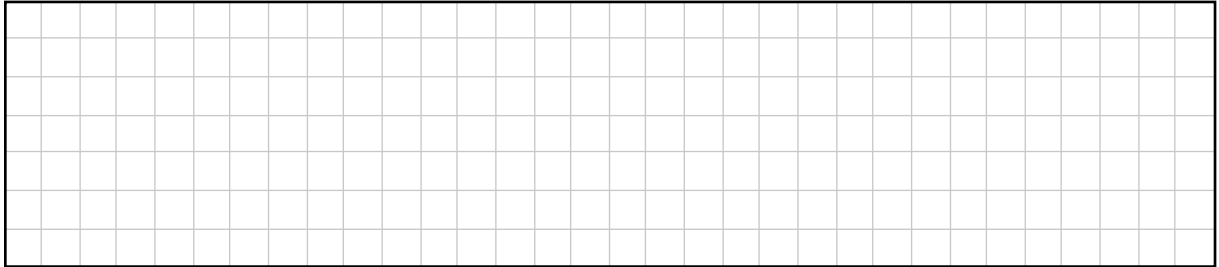
- **A**: all integer multiples of 4 (all numbers that can be written as  $4m$  where  $m \in \mathbb{Z}$ )
- **B**: all numbers that can be written as  $4^r$ , where  $r \in \mathbb{Q}$
- **D**:  $\mathbb{R} \setminus \mathbb{Q}$ , the set of irrational numbers.

The set  $A \cap D$  is shaded – there are no elements in this part of the Venn diagram.

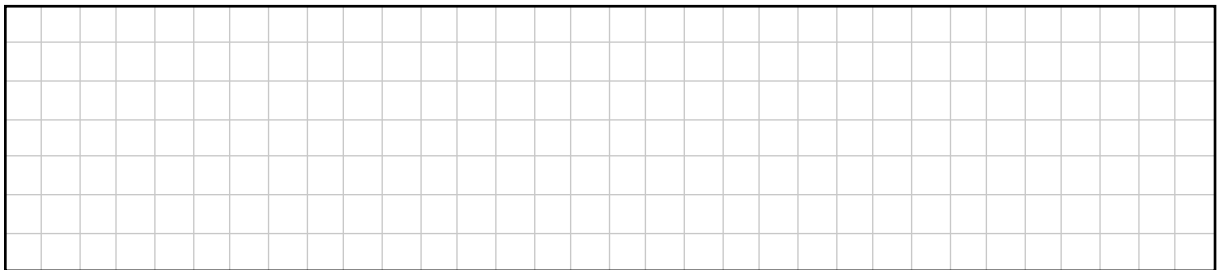


- (i) Write each of the following numbers in the correct region in the Venn diagram on the previous page.

64	$\frac{1}{16}$	$\sqrt{3}$	$\sqrt{2}$	12	1
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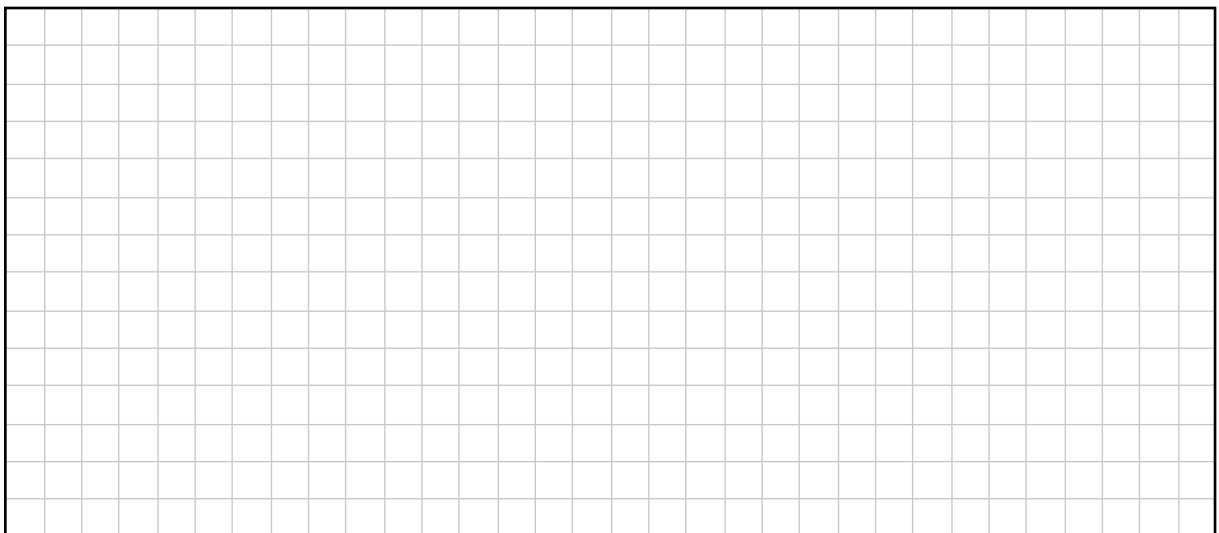
- (ii) Write down one element in  $\mathbb{R}$  that is **not** in any of the sets **A**, **B**, or **D**. Do **not** use any of the numbers from part **(b)(i)**.



- (c) For some real number  $k$ :

$$4^k = 9$$

Find the value of  $k$ . Give your answer in the form  $\frac{a}{\log_3 b}$  where  $a, b \in \mathbb{N}$  are constants.

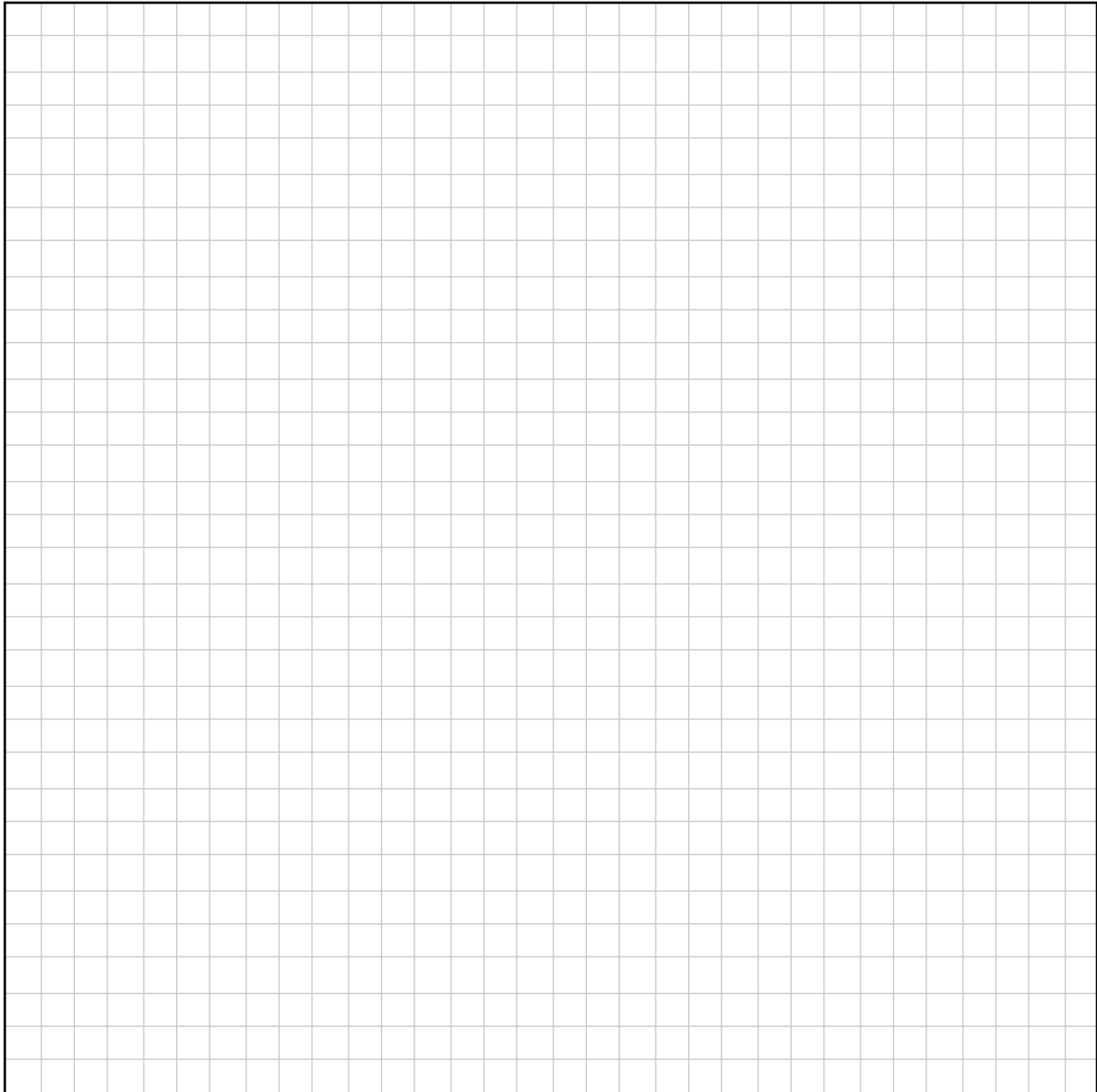


**Question 6**

**(30 marks)**

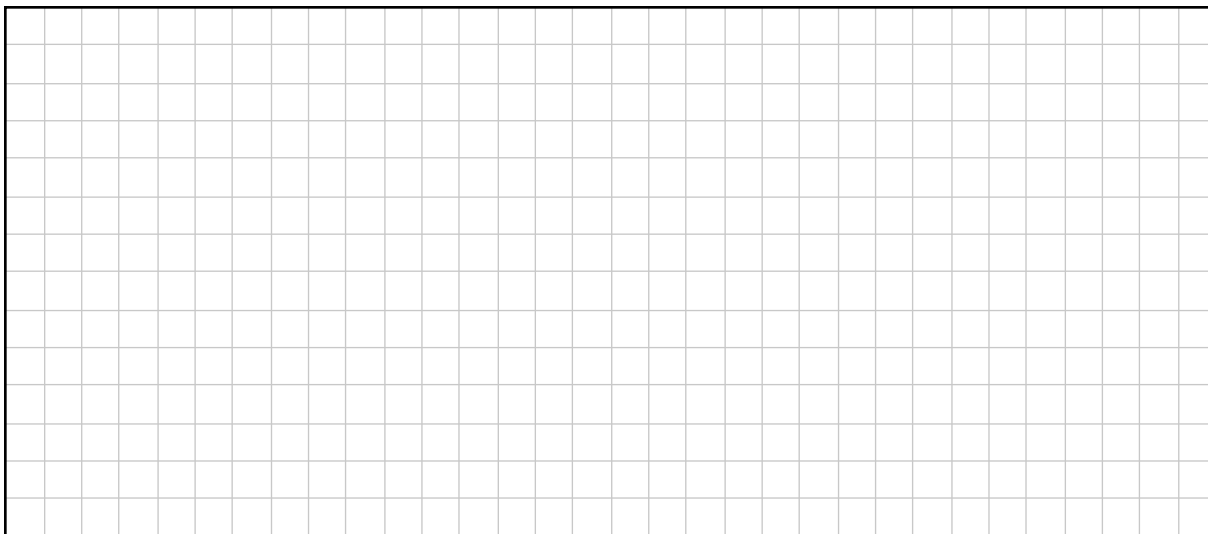
**(a)** Prove by **induction** that the following is true for all  $n \in \mathbb{N}$  and  $n \geq 7$ :

$$n! > 3^n$$



(b) Solve this equation in  $n \in \mathbb{N}$ :

$$\frac{(n+1)!}{(n-1)!} = 156$$



(c)  $T_1, T_2, T_3, \dots$  is a **geometric** sequence, where:

- the first term,  $T_1 < 2$
- the sum of the first two terms,  $T_1 + T_2 = 2$
- the sum of the first three terms,  $T_1 + T_2 + T_3 < 2$

Find one possible geometric sequence for which this is true.

State the first three terms of **your** sequence, **and** find the value of  $S_3 = T_1 + T_2 + T_3$ .

$T_1 =$  \_\_\_\_\_       $T_2 =$  \_\_\_\_\_       $T_3 =$  \_\_\_\_\_       $S_3 =$  \_\_\_\_\_

Answer **any three questions** from this section.

**Question 7****(50 marks)**

A water tank is in the shape of a cylinder.

Over a 4-minute period of time, the volume of water in the tank can be modelled by:

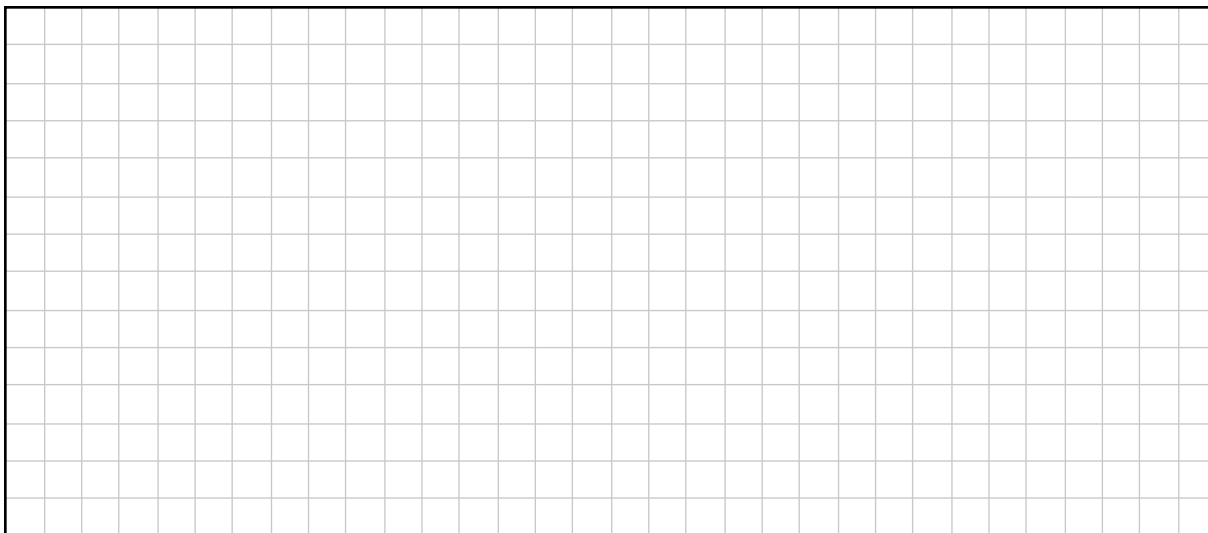
$$V(t) = 0.025 t^3 - 9 t^2 + 550.8 t + 68\,900$$

where  $V$  is in  $\text{cm}^3$  and  $t$  is the time, in seconds, for  $0 \leq t \leq 240$ ,  $t \in \mathbb{R}$ .

**(a)** Find the volume of water in the tank after 1 **minute**.

**(b)** Use differentiation to find the rate at which the volume of water in the tank is changing after exactly 150 seconds (that is, find  $\frac{dV}{dt}$  at this time). Give your answer in  $\text{cm}^3$  per second.

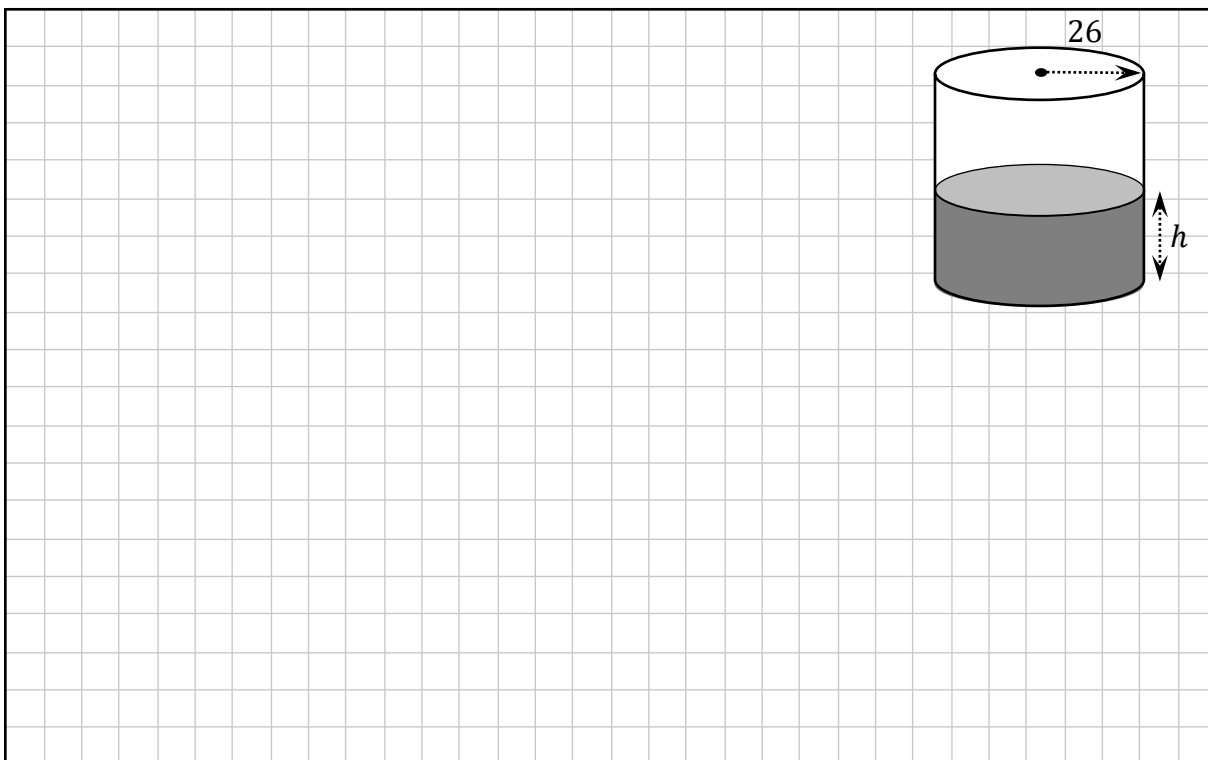
- (c) Find the range of values of  $t$  for which the volume of water in the tank is **decreasing**.



- (d) The water in the tank is in the shape of a cylinder with a constant radius of 26 cm and a height of  $h$  cm, where  $h \in \mathbb{R}$ ,  $h > 0$ .

Find  $\frac{dV}{dh}$  **and** hence, or otherwise, find the rate of change of the height of the water in the tank with respect to time after 150 seconds.

Give your answer in cm per second, correct to 2 decimal places.



*This question continues on the next page.*

Water is poured into another container. The function  $C(h)$  can be used to model the rate at which the height of this water is changing, in centimetres per second, when its height is  $h$  cm.

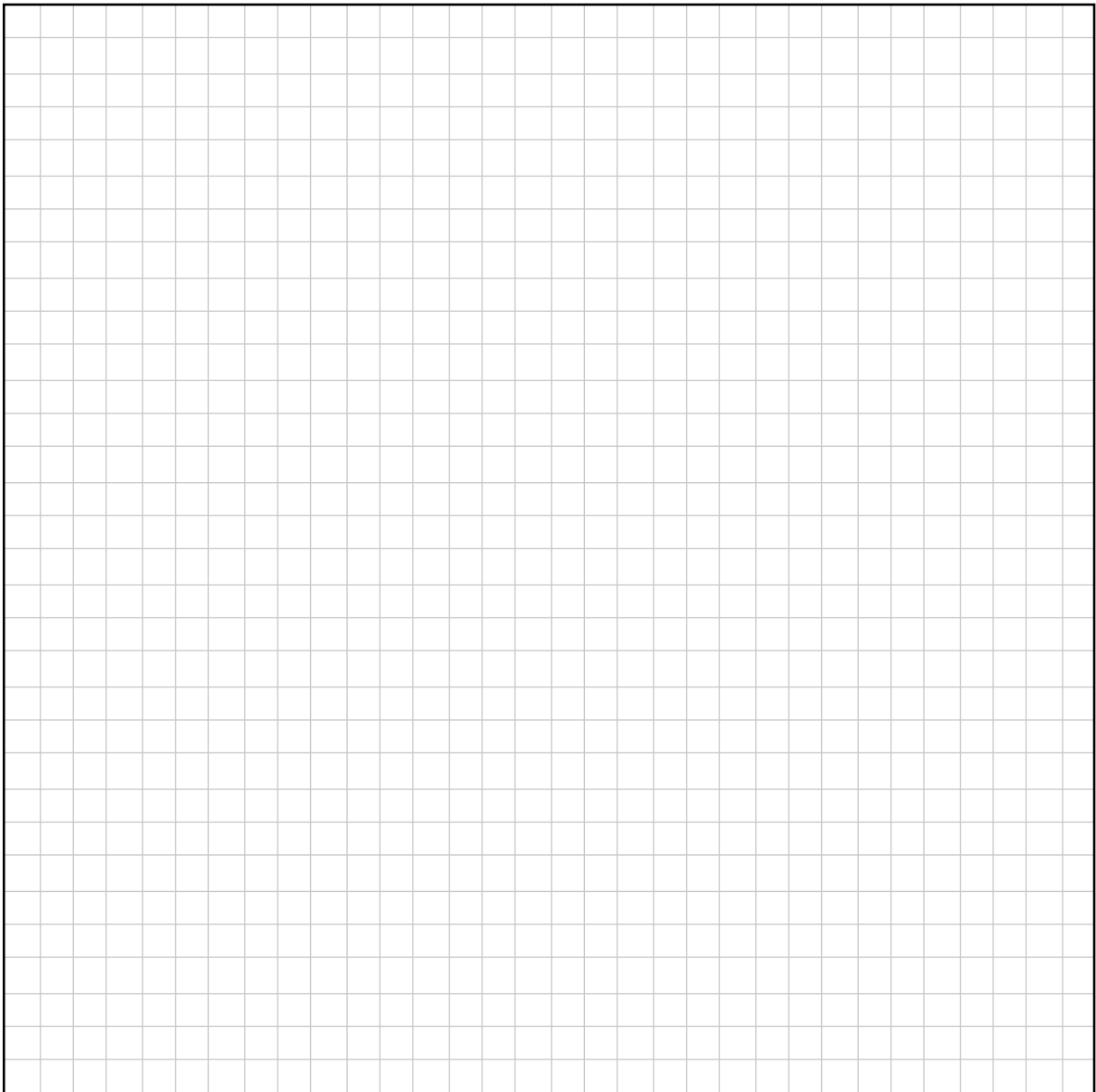
$C(h)$  is given by:

$$C(h) = \frac{20}{16\pi h - \pi h^2}$$

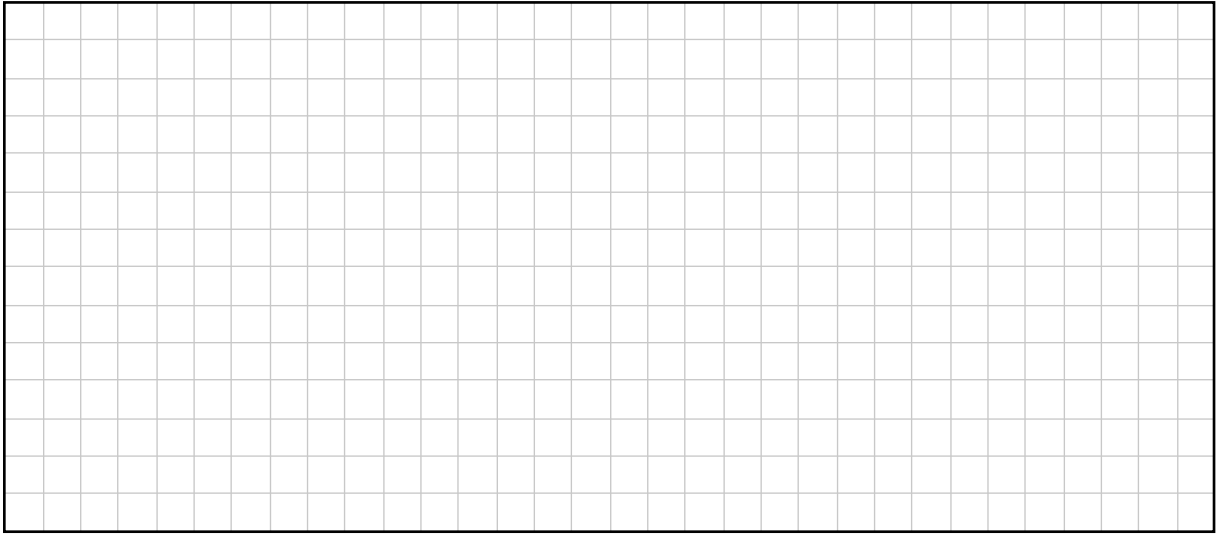
for a particular range of values of  $h \in \mathbb{R}$ , so that  $C(h)$  is a continuous function of  $h$  and has a local minimum value.

- (e) Find the local **minimum** value of  $C(h)$ .  
Give your answer in cm/second, correct to 1 decimal place.

Use differentiation in your solution.



- (f)  $C(h)$  is defined for all values of  $h \in \mathbb{R}$  for which  $0 < h < M$ , for a constant  $M \in \mathbb{R}$ .  
Find the **biggest** value that  $M$  can be.



**Question 8**

**(50 marks)**

Rían is tuning a piano. He has an app on his phone that can measure the frequency, in Hertz (Hz), of any note on the piano.

Rían calls one of the notes *Concert A*, and tunes it so that it has a frequency of 440 Hz. He then uses the following function  $F(x)$ , for  $x \in \mathbb{R}$ , to help him tune the other notes on the piano:

$$F(x) = 440 \times 2^{\frac{x}{12}}$$

For each note on the piano, Rían counts how many notes above (or below) *Concert A* it is.

Rían tunes the note that is  $x$  notes **above** *Concert A* to the frequency  $F(x)$ , for  $x \in \mathbb{N}$ .

He tunes the note that is  $x$  notes **below** *Concert A* to the frequency  $F(-x)$ , for  $x \in \mathbb{N}$ .

**(a)** Rían tunes the note that is 11 notes above *Concert A*.

By setting  $x = 11$ , use  $F(x)$  to find the frequency of this note, correct to the nearest whole number.

**(b)** Rían tunes the note that is 6 notes **below** *Concert A*.

Find the frequency of this note. Give your answer in Hz, correct to 4 significant figures.

- (c) Rían knows that the note that is seven notes above *Concert A* has a frequency of approximately 660 Hz.

Find the **actual** value of  $x \in \mathbb{R}$  for which  $F(x) = 660$ .

Give your answer correct to 2 decimal places.

- (d) Find an expression for  $F'(x)$ , the derivative of  $F$  with respect to  $x$ .

You do **not** need to simplify your answer.

*This question continues on the next page.*

(e)  $G(x)$  is another function that can be used to define the frequency of notes, for  $x \in \mathbb{R}$ .

For some number  $k \in \mathbb{R}$ :

$$G(k) = 197 \quad \text{and} \quad G'(k) = 5 \cdot 8.$$

(i) Using only this information, **estimate** the value of  $G(k + 2)$ .  
Show your working out.

(ii) The function  $G(x)$  is of the form

$$G(x) = A e^{bx}$$

where  $A$  and  $b$  are **positive** constants.

Based on this, state whether the estimate for  $G(k + 2)$  from part (e)(i) is too big or too small, compared to the actual value of  $G(k + 2)$ , or if we don't have enough information to be sure.

Justify your answer. It may be helpful to draw a diagram.

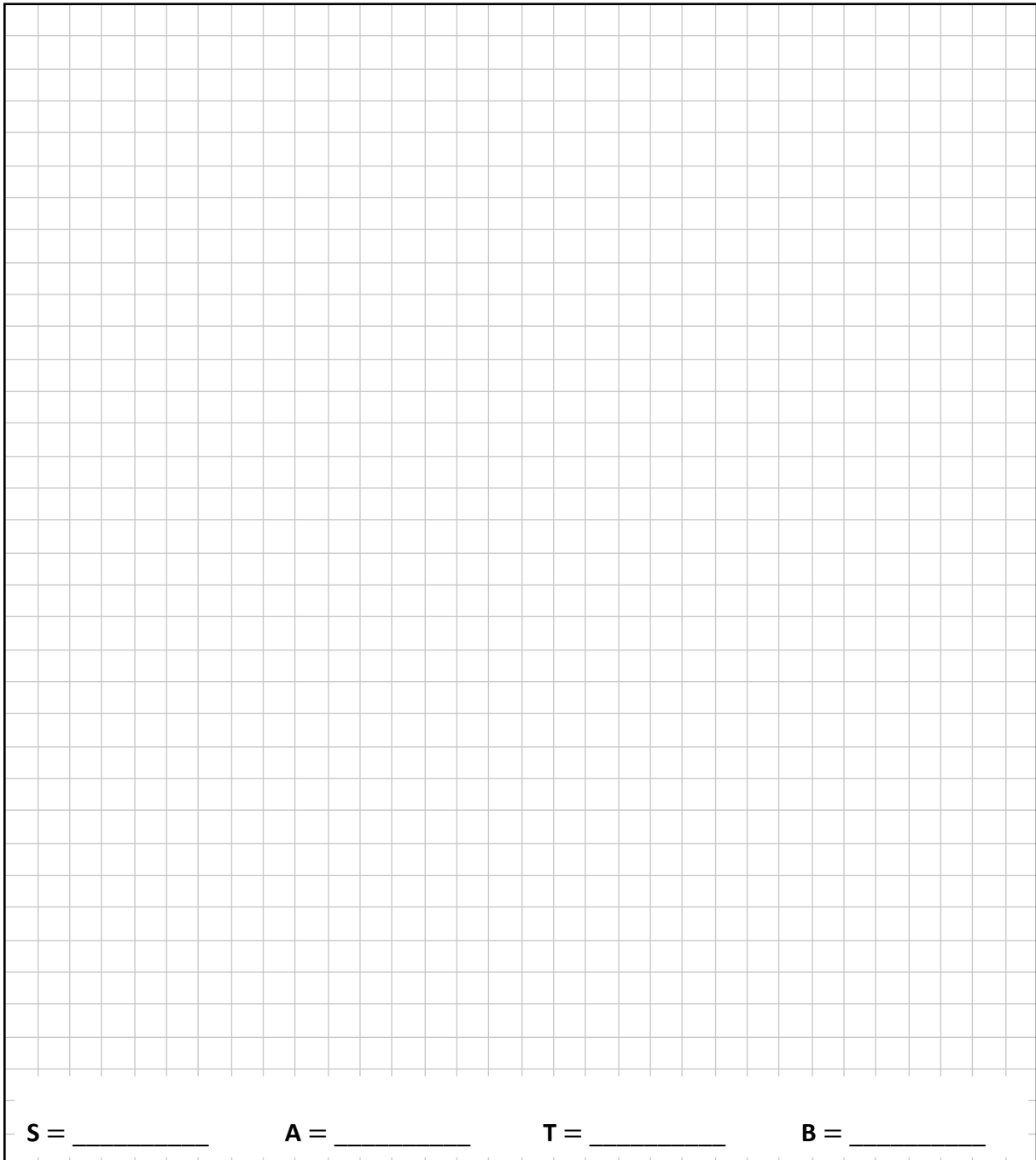
<b>Answer:</b>	the estimate is too big	the estimate is too small	we can't be sure
Tick (✓) <b>one</b> box only	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Justification:

(f) Each person in a choir is either a soprano (**S**), an alto (**A**), a tenor (**T**), or a base (**B**).  
In this choir:

- there are 384 people in total
- there are twice as many altos as there are sopranos
- there are 12 more tenors than there are bases
- the number of (sopranos plus altos) is three times the number of (tenors plus bases).

Work out the number of sopranos, of altos, of tenors, and of bases in the choir.



**S** = \_\_\_\_\_      **A** = \_\_\_\_\_      **T** = \_\_\_\_\_      **B** = \_\_\_\_\_



- (c) Find a formula, in terms of  $n \in \mathbb{N}$ , for the **total distance** (in metres) for Set  $n$ .  
Give your answer in the form  $an^2 + bn + c$ , where  $a, b, c \in \mathbb{R}$  are constants.

Total distance = \_\_\_\_\_

- (d) In Set  $h$ , shuttle 21 and shuttle 43 are the same length.  
Find the value of  $h$ .

$h =$  \_\_\_\_\_

- (e) Find an expression in  $k \in \mathbb{N}$  for the length of the  $k$ th shuttle in Set 18, where  $19 \leq k \leq 37$ .

Length = \_\_\_\_\_

*This question continues on the next page.*

As part of her training, Ella also lifts weights. The number of times in a row that she can lift any given weight is called the number of **reps** that she can do with that weight.

There are different formulas that Ella can use to estimate the heaviest weight that she could lift just once, called her one-repetition maximum ( $M$ ).

Two of these formulas are shown below. Each formula gives a value for  $M$  in kg, using the fact that Ella can do  $r$  reps with a weight of  $w$  kg.

<p>Epley formula:</p> $M = w \times \left(1 + \frac{r}{30}\right)$	<p>Brzycki formula:</p> $M = w \times \left(\frac{36}{37 - r}\right)$
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- (f) Find the **difference** in the value of  $M$  given by each formula for 5 reps ( $r = 5$ ). Give your answer as a fraction in terms of  $w$ .

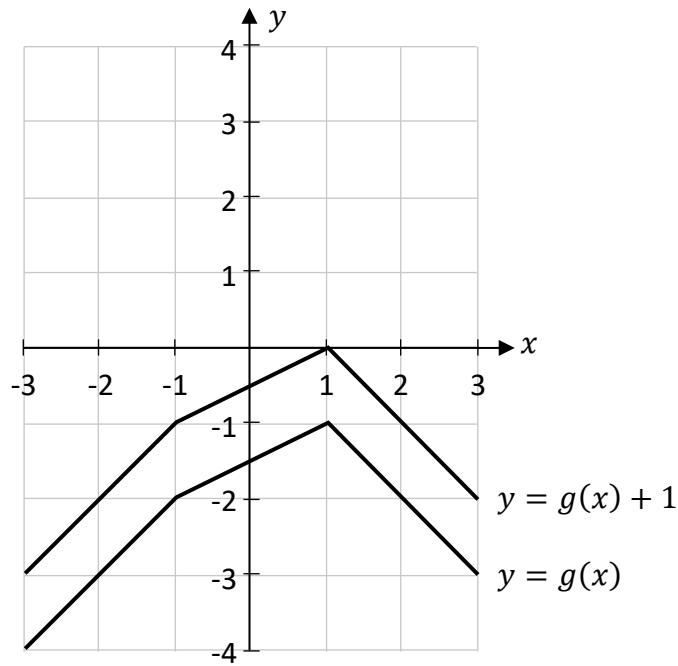
- (g) Solve an equation to find the number of reps ( $r$ ) for which the two formulas give the **same** value of  $M$ , for any given weight  $w$ .

**Question 10**

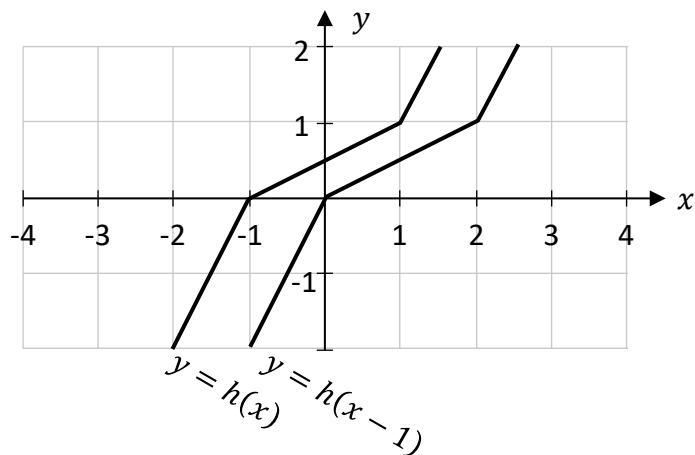
**(50 marks)**

Darek is looking at patterns in the co-ordinate plane.

- (a) (i) The diagram below shows the graphs of the functions  $g(x)$  and  $g(x) + 1$ .  
Draw and label the graph of  $y = g(x) + 3$  on the diagram below,  
using the same axes and scales.



- (ii) The diagram below shows the graphs of the functions  $h(x)$  and  $h(x - 1)$ .  
Draw and label the graph of  $y = h(x + 2)$  on the diagram below,  
using the same axes and scales.



*This question continues on the next page.*

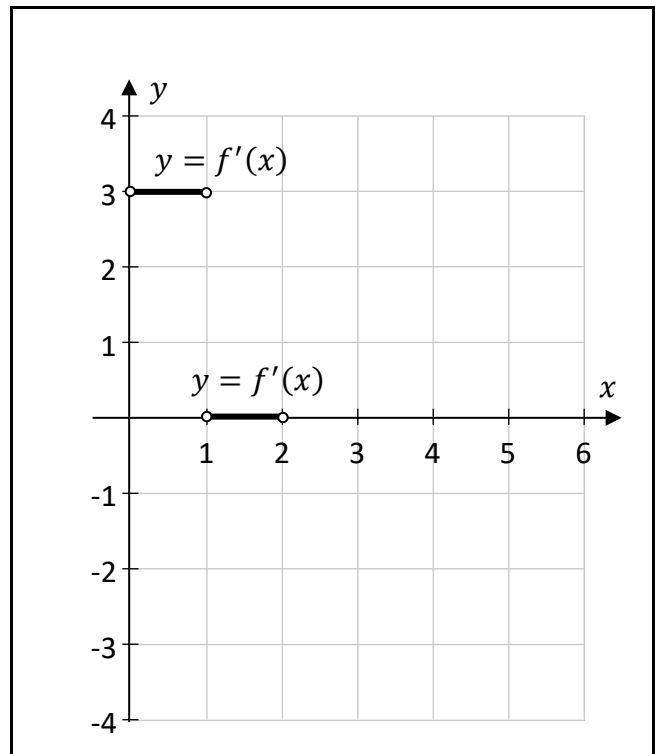
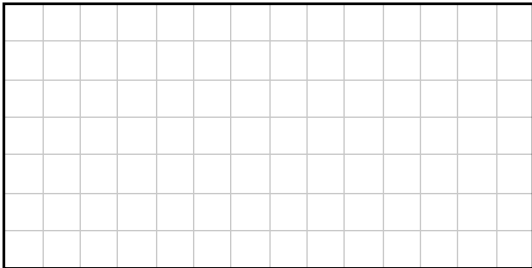


- (d) Based on the graph of  $f(x)$  on the previous page,  $f'(x)$ , the derivative of  $f(x)$ , is defined for  $x \in \mathbb{R}$  where  $0 < x < 6$  and  $x \neq 1, 2, 5$ .

The diagram on the right shows part of the graph of  $y = f'(x)$ .

It is equal to 3 for  $0 < x < 1$  and is equal to 0 for  $1 < x < 2$ .

Draw the graph of  $y = f'(x)$  in the diagram on the right, for  $2 < x < 6$ ,  $x \in \mathbb{R}$ ,  $x \neq 5$ .

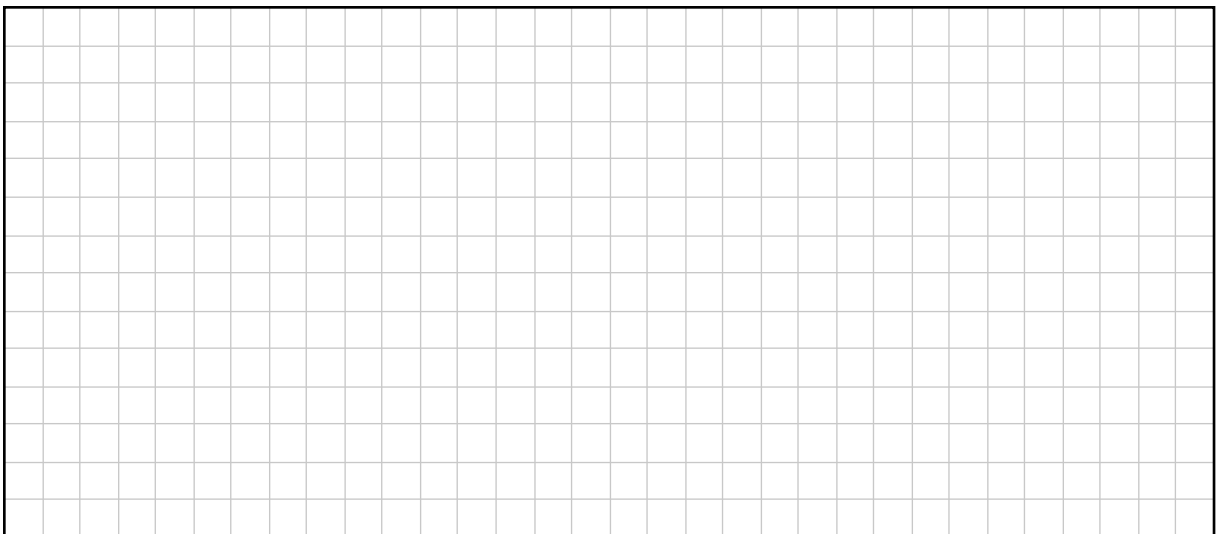


- (e)  $j(x)$  is a **different** function of  $x$ , with  $j(x) \geq 0$  for all  $x \in \mathbb{R}$ . For a constant  $a > 1$ :

$$\int_1^a j(x) dx = 200$$

Use this to find the value of the following in terms of  $a$  and  $b$ , where  $b > 0$  is a constant:

$$\int_1^a [j(x) + b] dx$$



*This question continues on the next page.*

(f) Darek is investigating if there are any functions  $h$  for which  $h(x + 1) = h(x) + 2$ .

He begins by looking at functions of the form:

$$h(x) = ax^2 + bx + c$$

where  $a, b, c \in \mathbb{R}$  are constants.

(i) Darek knows that  $h(x + 1) = ax^2 + 2ax + a + bx + b + c$ .

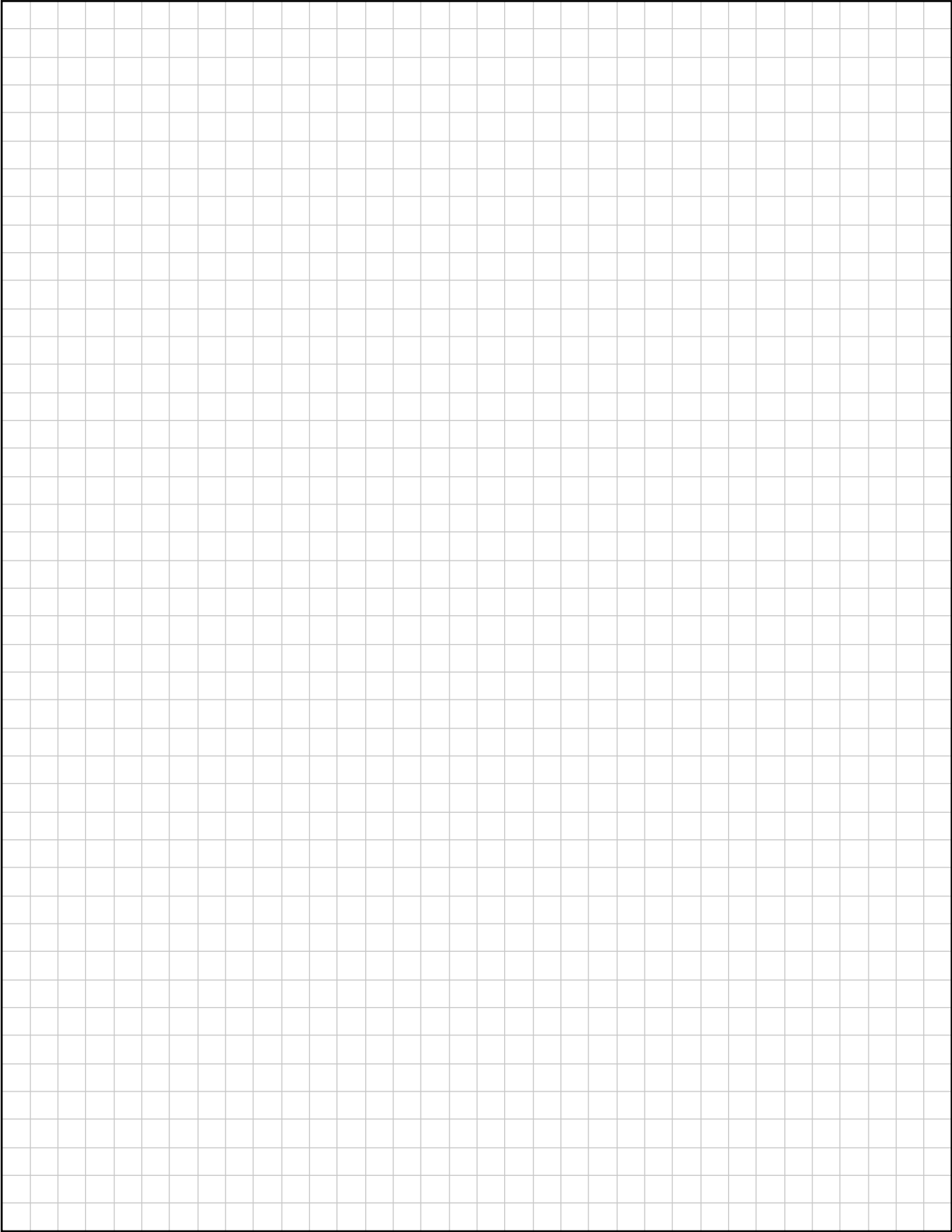
Use this to find the values of  $a$  and  $b$  that make the following true for all values of  $x$ :

$$h(x + 1) = h(x) + 2$$

$a =$  \_\_\_\_\_       $b =$  \_\_\_\_\_

(ii) State what information your values of  $a$  and  $b$  from part (f)(i) give about the graph of  $y = h(x)$ .

You may use this page for extra work.  
Label any extra work clearly with the question number and part.



Do not write on this page

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Leaving Certificate – Higher Level

## Mathematics - Paper 1

Friday 5 June

Afternoon 2:00 - 4:30